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WITH EXAMINATION QUESTIONS

SECOND EDITION

Graphical Analysis of Stresses

PART I

656A

INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.

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Study a few pages at a time—do not skip from one section of the Paper to another. If examples are given in the text, compare the solutions carefully with the rules, formulas, or other text matter relating to them. If there are EXAMPLES FOR PRACTICE, some or all may be worked, also; but this work need not be sent to the Schools for correction. If you meet with any difficulty, write us for help—using the "Information Blank." If there are any statements you do not understand, let us know, and we will explain them in detail. Pay particular attention to the definitions; a correct understanding of them is essential.

Review the entire subject; then write out your answers to the Examination Questions at the end of this Paper, and send your work to us for examination and correction. Your answers should occur in the same order as the questions are printed. Do not write out the questions—merely write their numbers. If you are unable to answer some question, write us, on an Information Blank, for assistance.

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INTERNATIONAL CORRESPONDENCE SCHOOLS

GRAPHICAL ANALYSIS OF STRESSES

(PART 1)

INTRODUCTION

1. The **graphical analysis of stresses** is the study, by means of diagrams, of the stability or equilibrium of structures and the relation between the external forces and the stresses created in the members of the frame. It is founded on the principle that any force may be designated, both as to direction and intensity, by a straight line by letting the direction of the line be identical with that of the force and adjusting its length according to an arbitrary unit adopted for the forces under consideration. The advantage existing in the use of graphical statics for the solution of stresses in framed structures is that when the principles are correctly applied no important error can exist, and though the stresses determined by this means will not be extremely accurate, they cannot be radically wrong. The approximations obtained by the diagrams give results as nearly accurate as the practical design of any member in the structure can be.

2. Definitions.—Before studying this subject the meaning of several terms should be thoroughly understood. Other terms, which require an extensive explanation, will be defined when first introduced.

A **force,** in graphical statics, is understood to be a weight or pressure applied in a certain direction at a particular point. It may be either an external or internal, or a concurrent or non-concurrent, or even a coplanar, or non-coplanar force.

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An **external force** on any structure represents either a weight or a reaction; for instance, the vertical dead or snow loads on a roof truss are external forces, as are also the reactions created by the resistance of the piers or abutments beneath the ends of the roof trusses.

An **internal force** is the stress created in any member of a framed structure. Sometimes it is denominated as a strain, but this is incorrect for strain is now generally understood to mean the distortion, or amount of change of form created in a piece of material by a stress or the force that must exist before the change of form is accomplished. In a roof truss, the internal forces are the compression in the rafter members and struts, and the tension in the tie-rods.

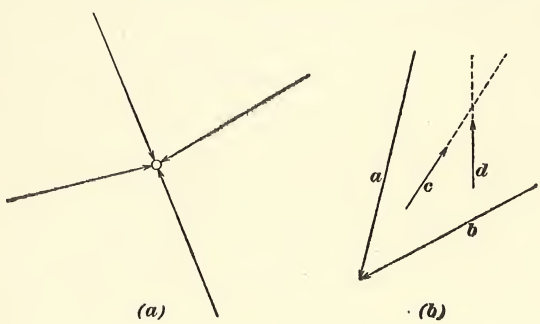


FIG. 1

Concurrent forces are those forces that act or meet at a single point, as in Fig. 1 (*a*).

Non-concurrent forces comprise any system of forces that do not meet at a single point, as in Fig. 1 (*b*). While *a* and *b* meet at a certain point, and *c* and *d* can be extended until they meet, the four forces cannot act at one point.

Coplanar and **non-coplanar forces** are the forces that act in the same plane, and in different planes, respectively. Fig. 2 (*a*) illustrates the first force and (*b*) the second.

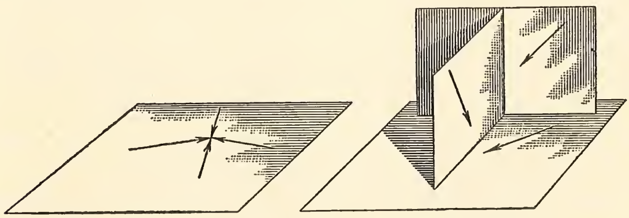
Equilibrium exists in any structure when there is no tendency for the structure to move; or if it is moving uniformly, there is no tendency for the rate or direction of the motion to change. In graphical statics, which term is

derived from the study of the equilibrium of structures, when a body is said to be in equilibrium, it is inferred that there is no tendency for the body or structural frame to change from a state of rest to one of motion; nor from a state of motion to one of rest. A body may be in equilibrium against motion either in a lateral direction or in a rotary direction.

Equilibrium of translation is a term that has been applied to a body in equilibrium against lateral motion.

Rotary equilibrium is applied to a body that is in equilibrium about a particular point; that is, the condition that exists in a body that has no rotary tendency about a fixed point.

Complete equilibrium can only be considered to exist when there is no tendency for a body toward translatory or



(a) (b)

FIG. 2

rotary motion; that is, when both the conditions of translatory equilibrium and rotary equilibrium are fulfilled.

A **couple** is the term given to two equal and opposing forces not coincident with respect to their line of action; or, in the terms of the definitions, two equal and opposing non-concurrent forces form a couple.

The **moment** of a force about any point not located in its line of action is its tendency to rotate about that point; the amount of the moment is obtained by multiplying the intensity, or amount, of the force by its lever arm, or the perpendicular distance from the line of action of the force to the point around which it tends to rotate, this point being termed the center of moments. The moment of any force is never expressed in either units of weight or units of

length, but by a combination of the two, as inch-pounds, foot-pounds, and foot-tons.

The **resultant** of any system of forces is any force that, by its direction and amount, will equal in its effect on a body or structure the effect of all the forces of the system. Any force that will destroy the action of all the forces of the system is equal and opposed to the resultant; such a force might be termed **resultant reaction.** Where the resultant of a system of forces equals zero, the system is in translatory equilibrium, while if the moment of the resultant about any point equals zero, the system is in rotary equilibrium. Where any system of forces is resolved into a couple, that is, two equal and opposing non-concurrent forces, the system cannot be in complete equilibrium and no single force will replace the two non-concurrent forces.

FUNDAMENTAL PRINCIPLES OF GRAPHICAL ANALYSIS

EFFECTS OF A FORCE

3. The **effect of a force** on a body may be compared with another force when the three following conditions are fulfilled in regard to both forces:

1. The point of application, or point at which the force acts on the body, must be known.
2. The direction of the force, or the straight line along which the force tends to move the point of application, must be known.
3. The magnitude, or value, of the force, when compared with a given standard, must be known.

The unit of magnitude of forces will be taken as 1 pound, throughout this Course, and all forces spoken of as a certain number of pounds.

4. The fundamental principles of the relations between force and motion, which were first stated by Sir Isaac Newton and are called "Newton's Three Laws of Motion," are as follows:

Law I.—*All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted on by some external force that compels a change.*

Law II.—*A force acting on a body in motion or at rest, produces the same effect whether it acts alone or with other forces.*

Law III.—*To every action there is always opposed an equal and contrary reaction.*

From the first law of motion, it is inferred that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted on by some force that compels a change.

The deduction from the second law is that, if two or more forces act on a body, their final effect on that body will be in proportion to their magnitude and to the directions in which they act. Thus, if the wind is blowing due west, with a velocity of 50 miles per hour, and a ball is thrown due north, with the same velocity, the wind will carry the ball just as far west as the force of the throw carried it north, and the combined effect will be to cause it to move northwest. The amount of departure from due north will be proportional to the force of the wind, and independent of the velocity due to the force of the throw.

FIG. 3

The third law states that action and reaction are equal and opposite. A man cannot lift himself by his boot straps, for the reason that he presses downward with the same force that he pulls upward; the downward reaction equals the upward action, and is opposite to it.

5. A force may be represented by a line; thus, in Fig. 3, let A be the point of application of the force, let the length of the line $A B$ represent its magnitude, and let the arrowhead indicate the direction in which the force acts, then the line $A B$ fulfils the three required conditions and shows the point of application, the direction, and the intensity of the force.

THE COMPOSITION OF FORCES

6. Parallelogram of Forces.—When two forces act on a body at the same time, but at different angles, their final result may be obtained as follows:

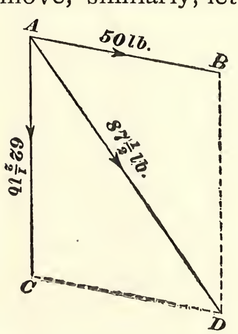


FIG. 4

In Fig. 4, let A be the common point of application of two forces, and let $A\ B$ and $A\ C$ represent the magnitude and direction of the forces. The final effect of the movement due to these two forces will be the same whether they act singly or together. For instance, let the line $A\ B$ represent the distance that the force $A\ B$ would cause the body to move; similarly, let $A\ C$ represent the distance that the force $A\ C$ would cause the body to move, when both forces were acting separately. The force $A\ B$, acting alone, would carry the body to B; if the force $A\ C$ were now to act on the body, it would carry it along the line $B\ D$, parallel to $A\ C$, to a point D, at a distance from B equal to $A\ C$. Join C and D, then $C\ D$ is parallel to $A\ B$, and $A\ B\ D\ C$ is a parallelogram. Draw the diagonal $A\ D$. According to the second law of motion the body will stop at D whether the forces act separately or together, but if they act together, the path of the body will be along $A\ D$, the diagonal of the parallelogram. Moreover, the length of the line $A\ D$ represents the magnitude of a force, which acting at A in the direction $A\ D$, would cause the body to move from A to D; in other words, $A\ D$ measured to the same scale as $A\ B$ and $A\ C$, represents the magnitude and direction of the combined effect of the two forces $A\ B$ and $A\ C$, and is called the resultant. Suppose that the scale used was 50 pounds to the inch, then, if $A\ B = 50$ pounds, and $A\ C = 62\frac{1}{2}$ pounds, the length of $A\ B$ would be $\frac{50}{50} = 1$ inch, and the length of $A\ C$ would be $\frac{62.5}{50} = 1\frac{1}{4}$ inches. If $A\ D$, or the resultant, measures $1\frac{3}{4}$ inches, its magnitude would be $1\frac{3}{4} \times 50 = 87\frac{1}{2}$ pounds. Therefore, a

force of $87\frac{1}{2}$ pounds, acting on a body at *A*, in the direction *A D*, will produce the same result as the combined effects of a force of 50 pounds acting in the direction *A B*, and a force of $62\frac{1}{2}$ pounds acting in the direction *A C*.

7. This method of finding the resulting action of two forces acting on a body at a common point, is correct for forces of any direction and magnitude. Hence, to find the resultant of two forces when their common point of application, their direction, and magnitudes are known:

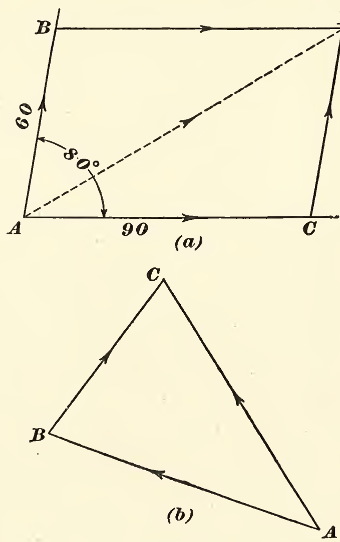


FIG. 5

Rule.—*Through an assumed point, draw two lines parallel with the direction of the two forces. With any scale, measure from the point of intersection, in the direction of the forces, distances corresponding to the magnitudes of the respective forces, and from the points thus obtained complete the parallelogram. Draw the diagonal of the parallelogram from the point of intersection of the two forces; this diagonal will be the resultant, and its direction will be away from the point of intersection of the two forces. Its magnitude must be measured with the same scale that was used to lay off the two forces.*

This method is called the **graphical method of the parallelogram of forces.**

EXAMPLE.—If two forces act on a body at a common point, both acting away from the body, and the angle between them is 80°, what is the value of the resultant, the magnitude of the two forces being 60 and 90 pounds, respectively.

SOLUTION.—Draw two indefinite lines having an angle of 80° between them. With any convenient scale, say 10 lb. to the inch, measure off $A B = 60 \div 10 = 6$ in., and $A C = 90 \div 10 = 9$ in., as shown in Fig. 5 (*a*). Through *B* draw *B D* parallel to *A C*, and through *C*

draw CD parallel to AB. Then draw AD, which will be the resultant; its direction is toward the point D, as shown by the arrow.

Measuring AD, its length is found to be 11.7 in. Hence, $11.7 \times 10 = 117$ lb. Ans.

8. Triangle of Forces.—The above example might also have been solved by the method called the **triangle of forces**, which is as follows: In Fig. 5 (*b*), suppose that the two forces AB and BC act separately, first from A to B, and then from B to C, in the direction of the arrows. Connect A and C; then AC is the resultant of the forces AB and BC. It will also be noticed in following the direction of the forces around the triangle, that the direction of the resultant AC is opposite to that of AB and BC. Hence, to find the resultant of two forces acting on a body at a common point, by the method of triangle of forces:

Rule.—*Draw the lines of action of the two forces as if each force acted separately, the lengths of the lines being proportional to the magnitude of the forces. Join the extremities of the two lines by a straight line, which will be the resultant; its direction will be opposite to that of the two forces.*

When the resultant is spoken of as being opposite in direction to the other forces around the polygon, it is meant that, starting from the point where the drawing of the polygon was commenced, and tracing each line in succession, the pencil will have the same general direction around the polygon as if passing around a circle, from left to right or from right to left, but the closing line or resultant must have an opposite direction; that is, the two arrowheads, the one on the resultant and the other on the last side, must point toward the intersection of the resultant and the last side.

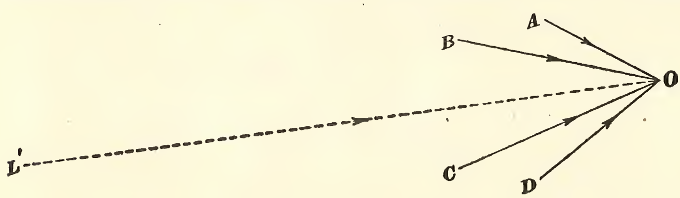
9. Resultant of Several Forces.—When three or more forces act on a body at a given point, their resultant may be found by the following rule:

Rule.—*Find the resultant of any two forces; treat this resultant as a single force, and combine it with a third force to find a second resultant. Combine this second resultant with a fourth force, to find a third resultant, etc. After all the forces have*

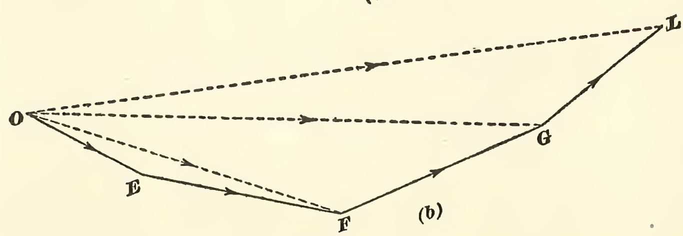
been thus combined, the last resultant will be the resultant of all of the forces, both in magnitude and direction.

Example.—Find the resultant of all the forces acting on the point O in Fig. 6, the length of the lines being proportional to the magnitude of the forces.

Solution.—Draw $O\,E$ parallel and equal to $A\,O$, and $E\,F$ parallel and equal to $B\,O$, then $O\,F$ is the resultant of these two forces, and its direction is from O to F, opposed to $O\,E$ and $E\,F$. Treat $O\,F$ as if $O\,E$ and $E\,F$ did not exist, and draw $F\,G$ parallel and equal to $O\,C$; $O\,G$ will be the resultant of $O\,F$ and $F\,G$; but $O\,F$ is the resultant of $O\,E$ and $E\,F$; hence, $O\,G$ is the resultant of $O\,E$, $E\,F$, and $F\,G$, and therefore of $A\,O$, $B\,O$, and $C\,O$. Likewise draw $G\,L$ parallel



(a)



(b)

Fig. 6

and equal to $D\,O$. Join O and L, and $O\,L$ will be the resultant of all the forces $A\,O$, $B\,O$, $C\,O$, and $D\,O$ (both in magnitude and direction), acting at the point O. If $L'\,O$ were drawn parallel and equal to $O\,L$, and having the same direction, it would represent the effect produced on the body by the combined action of the forces $A\,O$, $B\,O$, $C\,O$, and $D\,O$.

10. In Fig. 6, it will be noticed that $O\,E$, $E\,F$, $F\,G$, $G\,L$, and $L\,O$ are sides of a polygon $O\,E\,F\,G\,L$, in which $O\,L$, the resultant, is the closing side, and that its direction is opposed to that of all the other sides. This fact is made use of in what is called the **method of the polygon of forces.** To find the resultant of several forces acting on a body at the same point by this method:

Rule.—*Through any point, draw a line parallel to one of the forces, and having the same direction and magnitude. At the end of this line, draw another line, parallel to, and having the same direction and magnitude as a second force; at the end of the second line, draw a line parallel and equal in magnitude and direction to a third force. Thus continue until lines have been drawn parallel and equal in magnitude and direction to all of the forces.*

The straight line joining the free ends of the first and last lines will be the closing sides of the polygon; mark it opposite in direction to that of the other forces around the polygon, and it will be the resultant of all the forces.

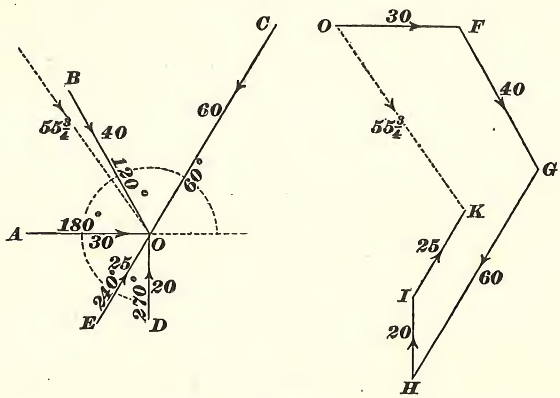


Fig. 7

Example.—If five forces act on a body at angles of 60°, 120°, 180°, 240°, and 270°, toward the same point, and their respective magnitudes are 60, 40, 30, 25, and 20 pounds, find the magnitude and direction of their resultant by the method of polygon of forces.*

Solution.—From a common point *O*, Fig. 7, draw the lines of action of the forces, making the given angles with a horizontal line through *O*, and mark them as acting toward *O*, by means of arrowheads, as shown. Choose some convenient scale, such that the whole figure may be drawn in a space of the required size on the drawing. Select any one of the forces, as *A O*, and draw *O F* parallel to it, and equal in length

*All the angles in the figure are measured from a horizontal line in a direction opposite to the movement of the hands of a watch, from 1° up to 360°.

to 30 lb. on the scale. It must also act in the same direction as *A O*. At *F*, draw *F G* parallel to *B O*, and equal to 40 lb. In a similar manner, draw *G H*, *H I*, and *I K* parallel to *C O*, *D O*, and *E O*, and equal to 60, 20, and 25 lb., respectively. Join *O* and *K* by *O K*, and *O K* will be the resultant of the combined action of the five forces; its direction is opposite to that of the other forces around the polygon *OFGHIK*, and its magnitude = $55\frac{3}{4}$ lb. Ans.

11. If the resultant *O K*, Fig. 7, were to act alone on the body in the direction shown by the arrowhead with a force of $55\frac{3}{4}$ pounds, it would produce exactly the same effect as

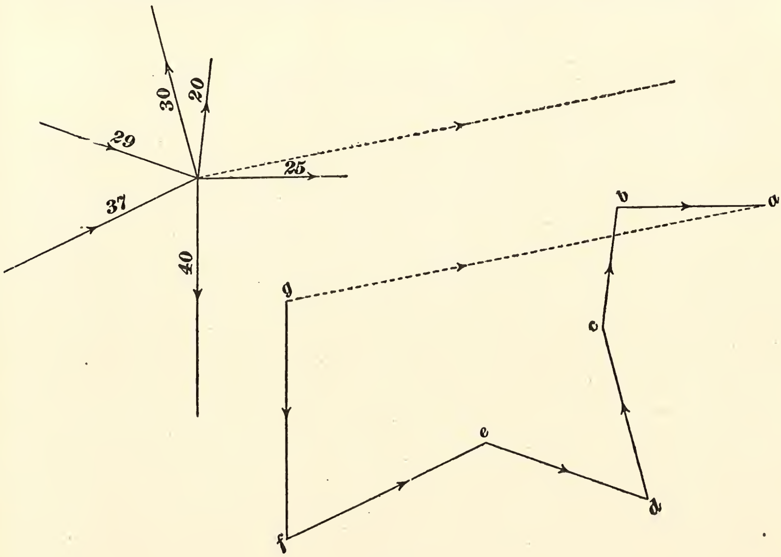


FIG. 8

the combined action of the five forces. If *OF*, *FG*, *GH*, *HI*, and *IK* represent the distances and directions that the forces would move the body, if acting separately, *OK* is the direction and distance of movement of the body when all the forces act together. It is evident, therefore, that any number of forces acting on a body at the same point, or having their lines of action pass through the same point, can be replaced by a single force (resultant), whose line of action shall pass through that point.

Heretofore, it has been assumed that the forces acted on a

single point on the surface of the body, but it will make no difference where they act, so long as the lines of action of all the forces intersect at a single point either within or without the body, only so that the resultant can be drawn through the point of intersection. If two forces act on a body in the same straight line and in the same direction, their resultant is the sum of the two forces; but if they act in opposite directions, their resultant is the difference of the two forces, and its direction is the same as that of the greater force. If they are equal and opposite, the resultant is zero, or one force just balances the other.

EXAMPLE.—Find the resultant of the forces whose lines of action pass through a single point, as shown in Fig. 8.

SOLUTION.—Take any convenient point *g*, and draw a line *gf*, parallel to one of the forces, say the one marked *40*, making it equal in length to 40 lb. on the scale, and indicate its direction by the arrowhead. Take some other force—the one marked *37* will be convenient; the line *fe* represents this force. From the point *e* draw a line parallel to some other force, say the one marked *29*, and make it equal in magnitude and direction to it. So continue with the other forces, taking care that the general direction around the polygon is not changed. The last force drawn in the figure is *ab*, representing the force marked *25*. Join the points *a* and *g*; then, *ag* is the resultant of all the forces shown in the figure. Its direction is from *g* to *a*, opposed to the general direction of the others around the polygon. It does not matter in what order the different forces are taken, the resultant will be the same in magnitude and direction, if the work is done correctly.

These various methods of finding the resultant of several forces are all grouped under one head: The Composition of Forces.

THE RESOLUTION OF FORCES

12. Since two forces can be combined to form a single resultant force, a single force may also be treated as if it were the resultant of two or more forces, whose action on a body will be the same as that of the single force. Thus, in Fig. 9, the force *O A* may be resolved into two forces, *O B* and *B A*, whose directions are opposed to *O A*. If the force *O A* acts on a body, moving or at rest on a horizontal plane, and the resolved force *O B* is vertical, and *B A*

horizontal, $O B$, measured to the same scale as $O A$, is the magnitude of that part of $O A$ that pushes the body downwards, while $B A$ is the magnitude of that part of the force $O A$, which is exerted in pushing the body in a horizontal direction. $O B$ and $B A$ are called the **components** of the force $O A$, and when these components are vertical and horizontal, as in the present case, they are called the *vertical component* and the *horizontal component* of the force $O A$. These components may be drawn in any direction and the angle at their intersection is not necessarily a right angle.

13. It frequently happens that the position, magnitude, and direction of a certain force are known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 9, assume that $O A$ represents, to some scale, the magnitude, direction, and line of action of a force acting on a body at A, and that it is desired to know what effect $O A$ produces in the direction $B A$, which may be any direction. To find the value of the component of $O A$ that acts in the direction $B A$, it is necessary to employ the following rule:

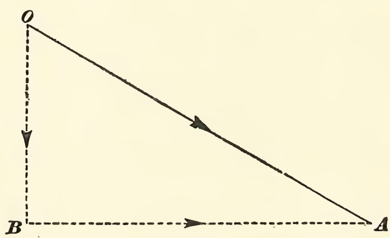


FIG. 9

Rule.—*From one extremity of the line representing the given force, draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force, draw a line perpendicular to the component first drawn, and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will be the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose that $O A$, Fig. 9, represents a force acting on a body resting on a horizontal plane, and that it is desired to know what vertical pressure $O A$ produces on the body.

Here the desired direction is vertical; hence, from one extremity, as O, draw OB parallel to the desired direction (vertical in this case), and from the other extremity draw AB perpendicular to OB, and intersecting OB at B. Then OB, when measured to the same scale as OA, will be the value to the vertical pressure produced by OA.

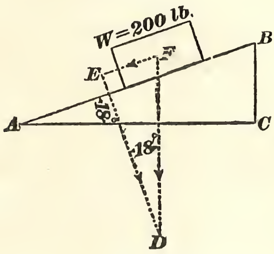


FIG. 10

EXAMPLE.—If a body weighing 200 pounds rests on an inclined plane whose angle of inclination to the horizontal is 18°, what force does it exert perpendicular to the plane, and what force does it exert parallel to the plane, tending to slide downwards?

SOLUTION.—Let $A\ B\ C$, Fig. 10, be the plane, the angle A being equal to 18°, and let W be the weight. Draw a vertical line $FD = 200$ lb., to represent the magnitude of the weight. Through F, draw FE parallel to AB, and through D draw DE perpendicular to EF, the two lines intersecting at E. FD is now resolved into two components, one FE tending to pull the weight downwards, and the other ED acting as a perpendicular pressure on the plane.

On measuring FE with the same scale by which the weight FD was laid off, its intensity is found to be about 61.8 lb., and the perpendicular pressure ED on the plane is found to measure 190.2 lb. Ans.

EQUILIBRIUM

14. When a body is at rest, all of the forces that act on it must balance one another; the forces are then said to be in equilibrium. The most important of the forces acting on the body is gravity, which acts on every particle. But a force that must be considered when determining the equilibrium of framed structures is the wind pressure.

A body is in *stable equilibrium* when, if slightly displaced from its position of rest, the forces acting on it tend to return it to that position; for example, a cube, a cone resting on its base, a pendulum, etc.

A body acted on by a system of forces is in *unstable equilibrium* when the application of a small force is sufficient to produce motion; for example, a cone standing on its apex, an egg balanced on end, etc.

Since two kinds of motion may be produced in a body acted upon by external forces, the following conditions must be fulfilled in order that a body be in equilibrium:

1. The resultant of all the forces tending to move the body in any direction must be zero.

2. The resultant of all the forces tending to turn the body about any center must be zero.

But, if either of the two following conditions prevails the body will be in unstable equilibrium or unrest:

1. If the forces acting on a body create or influence motion of the body in the direction of the line of action of the force.

2. If the force acting on the body tends to move or rotate the body around some fixed point, which point is always necessarily outside the line of action of the force.

EQUILIBRIUM AT THE JOINTS OF A STRUCTURE

15. The several forces acting at any joint of a framed structure must theoretically be concurrent, and a single joint can, in consequence, be subjected only to translatory motion. But, where several forces meet at a joint in any stable structure there must be no translatory motion, for if the joint could move in any direction it would fail and cause the destruction of the frame.

In Fig. 11 (*a*) are shown five concurrent forces that act in the directions indicated by the arrows. By drawing the stress diagram for these forces, as shown at (*b*), their resultant is determined by the line *b c*, which force is the combined effect, both in direction and in intensity, of all of the concurrent forces acting at the joint *c*. Since this line *b c* is the resultant of the system of forces, it is evident that if no other force is substituted at the joint, it will move in the oblique direction indicated by the arrow on this line in the stress diagram. In order to produce equilibrium in the joint, a force *c b* in (*a*) acting in opposition to *c a*, and of the same amount, must be applied to the joint *c*, as shown. Consequently, the joint is in unstable equilibrium and can never be in equilibrium of translation until the points *c* and *b* in the stress diagram coincide.

The stress diagram of the five concurrent forces, acting on the joint shown in Fig. 11 (c), is shown in (d). It will be observed that there is no resultant, but that the polygon closes at the point c, the end of the last force $f\,c$ coinciding with the beginning of the first force $a\,c$. It is evident, therefore,

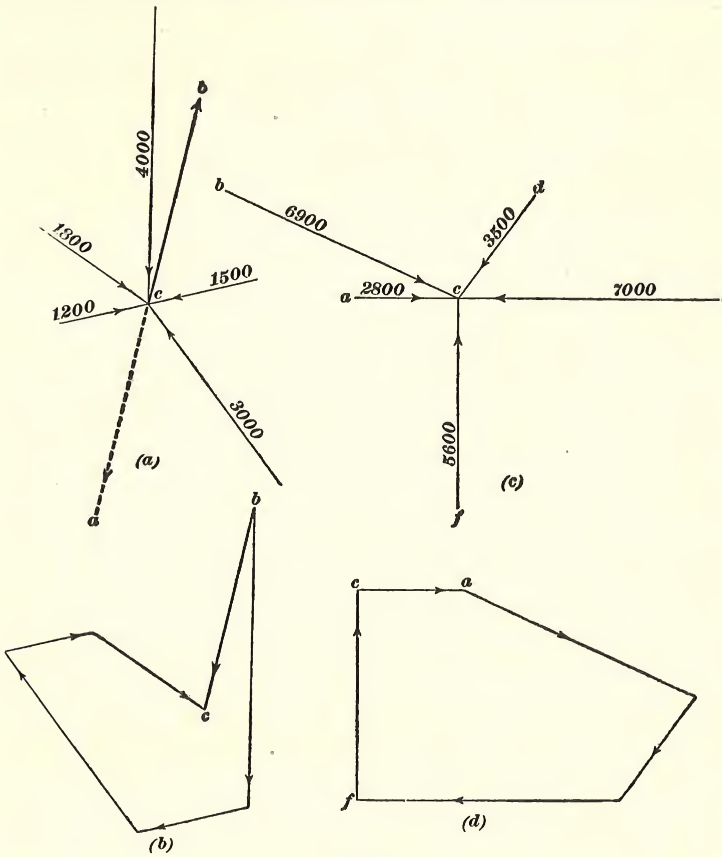


Fig. 11

since the resultant of all the forces about the joint c is zero, that no force need be substituted in the frame diagram to produce equilibrium of translation, and in consequence the joint in the structure will be stable and no failure of the frame, through weakness at this joint, can exist.

16. In order to analyze the conditions that create equilibrium of translation in any system of concurrent forces, resolve each of the forces of the system shown at Fig. 11 (c) into vertical and horizontal components, as shown in Fig. 12, by the method explained under The Resolution of Forces. The forces $a\,c$ and $e\,c$, Fig. 11 (c), are horizontal and $f\,c$ is vertical, but the forces $b\,c$ and $d\,c$ are oblique and may be resolved into vertical and horizontal components, as shown by y and y_1, x and x_1, respectively, Fig. 12. By scaling the horizontal components x and x_1, it is found that $x = 2{,}100$ pounds and $x_1 = 6{,}300$ pounds, and it will also be observed that the direction of the component x is opposed to the

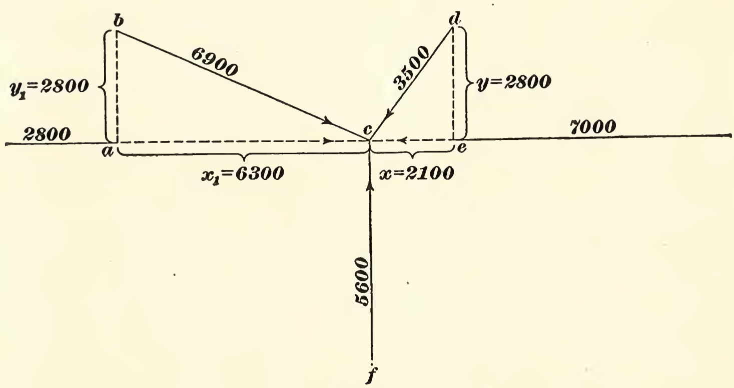


Fig. 12

direction of the force $a\,c$, and x_1 is opposed to the force $e\,c$. By studying the diagram, Fig. 12, it will be observed that the horizontal forces acting toward the left are exactly equal to the horizontal forces acting toward the right, and that they act and react so as to completely annul all action in a horizontal direction. In other words, the algebraic sum of all of the horizontal forces acting at the joint is zero, or, as it might be stated, $x + ec = x_1 + ac$ and $(x + ec) - (x_1 + ac) = 0$.

Again, the vertical components y_1 and y of the forces $b\,c$ and $d\,c$ both act in opposition to the force $f\,c$ and the algebraic sum of all the vertical forces is equal to zero, for the sum of y and y_1 is equal to 5,600, as may be proved by

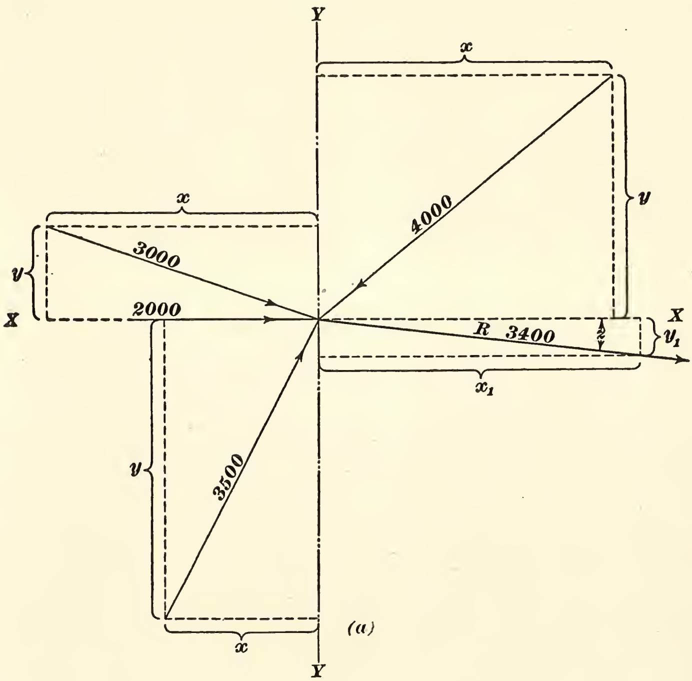
scaling the diagram. The algebraic sum of all the vertical forces at the joint is consequently equal to zero. From these deductions the condition of equilibrium at a joint may therefore be expressed by the following rule:

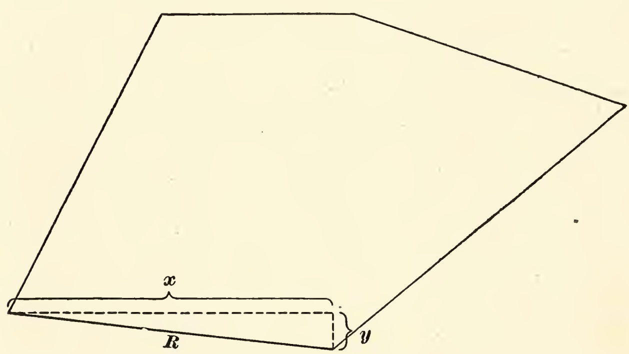
Rule.—*In order that any joint in a frame, or any system of concurrent forces, shall be in equilibrium of translation, the algebraic sum of all the horizontal and vertical forces and all the horizontal and vertical components of the oblique forces shall equal zero.*

17. The conclusion that in order to have equilibrium of translation the algebraic sum of the several forces at a joint must equal zero, can be reached in another way. For instance, it has been stated and proved that the resultant of any system of concurrent forces must equal zero and it is evident that when all of the forces about a joint are resolved into their horizontal and vertical components, the final resultant is the hypotenuse of the triangle formed by the resultants of the components. In Fig. 13 (*a*), the several oblique forces have been resolved into their components x, x, x, and y, y, y, and x_1 and y_1 are the algebraic sums of these components, as may be proved by scaling the diagram. By laying off x_1 and y_1, as indicated, and drawing the hypotenuse R, the resultant is obtained. This resultant is therefore $\sqrt{\Sigma x^2 + \Sigma y^2}$, in which x and y equal, respectively, the horizontal and vertical components of each force about the joint. The components x_1 and y_1 may also be obtained directly from the stress diagram, Fig. 13 (*b*), by resolving the resultant R into its vertical and horizontal components.

The direction of the resultant R can be determined by obtaining the tangent of the angle that R makes with the horizontal or vertical components. The tangent of the angle marked z in Fig. 13 (*a*), is equal to $\frac{y_1}{x_1}$. Where, therefore, the vertical and horizontal components of each of the forces are considered, their direction being represented by V and H, respectively,

$$\tan z = \frac{\Sigma y}{\Sigma x}, \text{ or } \frac{\Sigma\, V \text{components}}{\Sigma\, H \text{components}}$$





(b)

Fig. 13

But, from a previous statement and deduction, the resultant R of any system of concurrent forces in equilibrium must equal zero, so that $R^2 = (\Sigma H \text{ components})^2 + (\Sigma V \text{ components})^2 = 0$. R having an inappreciable amount, consequently, has no direction and it is conclusively shown that for equilibrium of translation the sum of the horizontal and vertical components must equal zero. This condition must therefore exist at every joint in a framed structure in order to secure the equilibrium or stability of the entire structure.

EQUILIBRIUM OF THE ENTIRE FRAME

18. Any framed structure, besides having each joint at which concurrent forces occur in equilibrium, must be in equilibrium with regard to the action of the external forces, which are usually non-concurrent. The external forces on any frame, such as a roof truss, are the loads and their reactions. On a roof truss, or other exposed structure, the external forces consist of the vertical loads due to the weight of the structure and snow, and the horizontal or oblique forces due to the wind pressure. The reactions occur at the abutments or supports of the structure and act in opposition to the loads, to hold the structure in equilibrium. It is sometimes necessary to introduce in the analysis of stresses imaginary or assumed reactions and forces that replace the resistance to bending offered by some member; this is due to the fact that a transverse stress cannot be shown in a diagram in conjunction with direct stresses.

Fig. 14 (*a*) shows an iron bracket resting on the ledge *a*, tied into the wall at *b*, and supporting, at its end, a weight W. As the external, or non-concurrent, forces alone are to be taken into account, the members of the bracket may be disregarded and the structure considered as a solid triangular body held in equilibrium by the four forces W, C, D, and E, as shown in (*b*). The force W acts vertically while C, which is a tensile stress, acts horizontally; D and E may be regarded as reactions. For the frame to be in equilibrium, the sum of the vertical forces must equal zero, as must also the sum

of the horizontal forces. W is known, both in direction and intensity, and since E is negative with respect to the force W, their algebraic sum must be $W - E = 0$; therefore, $W = E$. Since the resultant of the horizontal forces must equal zero, $C - D = 0$, or $C = D$, thus proving that the triangular frame is in translatory equilibrium; that is, there is no tendency for the frame to move laterally in any direction.

The conditions of complete equilibrium are not fulfilled unless equilibrium of both translation and rotation exists about any point of the structure. Assume the point c as the center of moments. The force C acts about this point with a lever arm equal to y and the weight W tends to rotate in the opposite direction with a lever arm equal to x. Since the forces E and

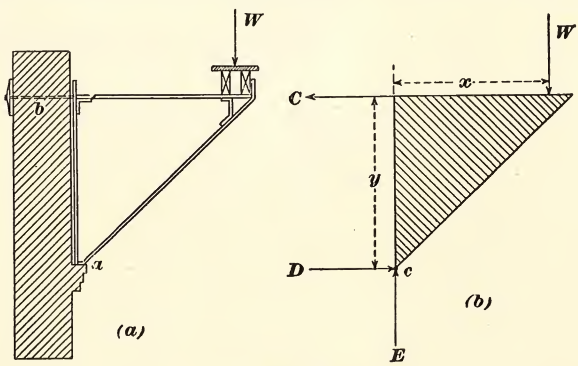


FIG. 14

D intersect at the center of moments c, there is no tendency for them to rotate the structure about this point, for their lever arm is zero; so that $D \times 0 = 0$, as does $E \times 0 = 0$. $W \times x$ must equal $C \times y$ in order to produce rotary equilibrium; therefore, $W \times x - C \times y = 0$. The algebraic sum of the moments of all the forces acting about c and tending to rotate the triangular frame is consequently equal to zero.

THE FORCE AND EQUILIBRIUM POLYGON

19. Since, in order for any structure to be in equilibrium the algebraic sum of the forces acting on the structure must equal zero, when a structure supports a number of

parallel loads, the reactions at the ends of the structure, which are coincident with the line of action of the loads, must equal, when added together, the sum of the loads.

If the loads are not all parallel, but exert their forces along different lines of action, and the reactions in consequence do not coincide with the line of action of the several forces, the sum of the reactions does not equal the sum of the loads; but, in order for the structure to be in equilibrium, the sum of the vertical and horizontal components of all the forces must equal the sum of the vertical and horizontal components of the reaction.

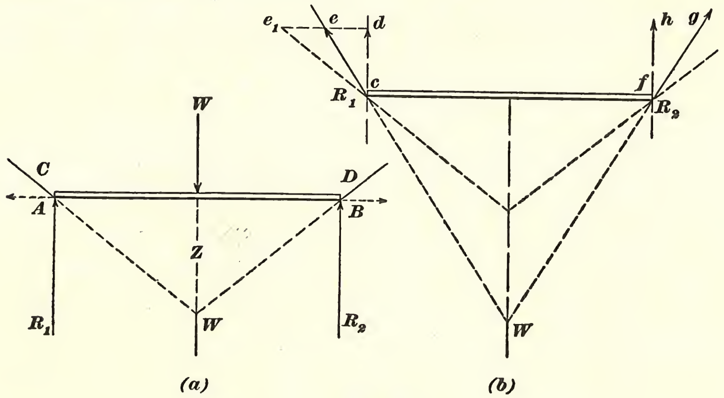


FIG. 15

Assume that in Fig. 15 (a), a beam loaded at the center with the weight W is held in equilibrium by the reactions R_1 and R_2; or, designating them by the usual system of notation, the forces $A Z$ and $B Z$. Suppose, now, that instead of a beam that has a certain transverse strength, which cannot be represented graphically, the load W is supported by a flexible cord attached at R_1 and R_2. This cord might have any length and consequently might drop any distance below the beam, but no matter what this drop may be, the vertical reactions R_1 and R_2 will always be the same, for the sum of R_1 and R_2 equals the sum of the vertical components of all of the forces acting in one direction on the cord, while W, or the amount of the load, must be equal and act in the opposite

direction to fulfil the condition that the algebraic sum of the vertical loads shall equal zero. Besides the reactions R_1 and R_2, in order to create equilibrium in the cord supporting the load W, there must be horizontal forces A and B at each end of the cord to prevent their approaching each other. These two forces could as well be supplied by a horizontal strut extending between the ends of the cord, and could be considered as the compressive strength of the beam.

It is evident, therefore, since there are horizontal and vertical forces at each end of the beam, that a single force equal to their resultant could be applied to the ends of the cord in order that equilibrium might be maintained. These oblique resultants C and D of the vertical and horizontal components must, therefore, in order that they may alone create equilibrium, have their line of action coincident with the direction of the cord, as shown. Assume that the cord, instead of occupying the position designated in Fig. 15 (a), is considerably longer and that the weight is applied at twice the distance below the beam, as in Fig. 15 (b). The directions of the oblique reactions coincident with the present direction of the cord are indicated by the lines $c\,e$ and $f\,g$. But it was stated that the vertical reactions, which are the vertical components of these oblique forces and are represented by the vertical lines $c\,d$ and $f\,h$, Fig. 15 (b), must always be of the same amount when the load W is located in the center of the cord; that is, their sum must equal the load. It is evident, then, that if the vertical component at R_1 is laid off from c to d, and $d\,e$ is drawn horizontally from d, $d\,e$ will equal the horizontal component of the oblique force at this abutment. It will also be noticed that when the oblique reaction C, Fig. 15 (a), is designated by the dotted line $c\,e_1$, Fig. 15 (b), $d\,e_1$ is the horizontal component for the oblique reaction at R_1. As seen in Fig. 15 (b), the horizontal reaction of the cord having the lesser drop is much greater than the horizontal reaction of the cord having the greater drop, and it is therefore evident that as the drop of the cord becomes greater, the horizontal component of the oblique force becomes less. Likewise, that the greater the drop of the cord, the less will

be the oblique reaction until when the oblique reaction approaches verticality the thrust or horizontal force will become zero while the reaction will equal one-half of the load.

20. Assume that to the beam shown in Fig. 16 (*a*) is applied a load *W* equal to 1,000 pounds. Lay off in the

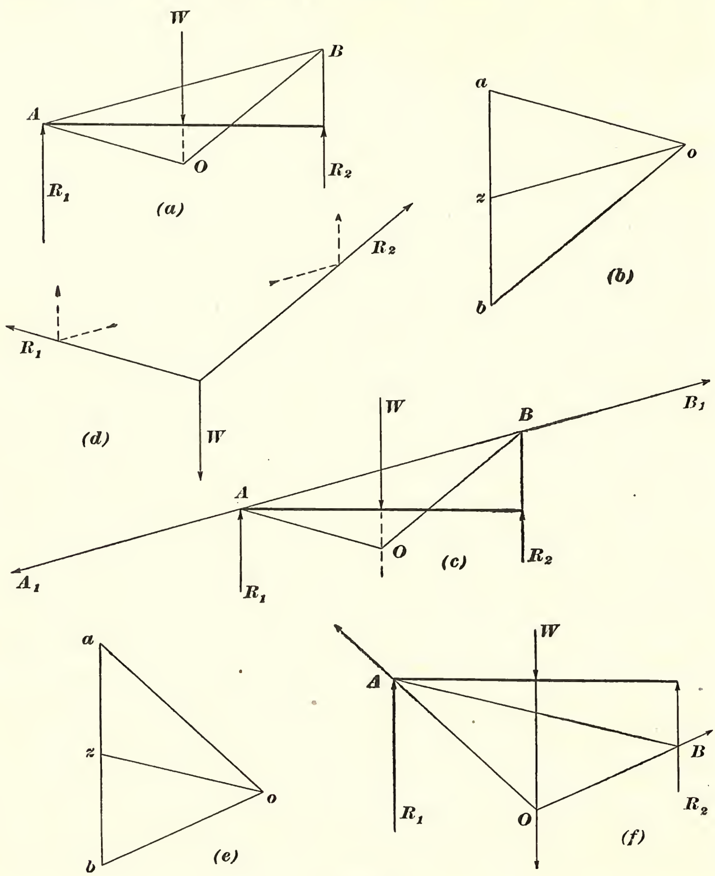


FIG. 16

diagram at (*b*) a line *a b* equal, by scale, to 1,000 pounds. Choose any point, or pole, such as *o*, and draw lines from *a* and *b* to this point. From the point *A* in (*a*), extend a line *A O* parallel with *a o* in (*b*). Extend the line of action

of the force W acting on the beam until it intersects this line just drawn at the point O. From the point O thus found, draw a line $O\,B$ parallel with $o\,b$ in (b) and designate the point of intersection of this line and the line of action of the reaction R_2, as B. Connect the points A and B just found in this diagram, and returning to the diagram (b), extend from the pole, or point o, a line coincident in direction with the line $A\,B$ in (a). Where this line intersects the line $a\,b$, designate the point as z. Fig. 16 (b) is known as the *force polygon*, while Fig. 16 (a) is known as the *equilibrium*, or *funicular*, *polygon*.

On analyzing these diagrams, it is found that the distances $b\,z$ and $z\,a$ are equal and since $a\,b$ represents the amount of the load and $b\,z$ and $z\,a$ are one-half of the load, it is evident that these two forces represent, in amount and direction, the reactions R_1 and R_2. No matter where the pole o had been chosen or assumed, the result would have been the same, for the line $o\,z$ is always drawn parallel with the line $A\,B$ found in the equilibrium polygon. On scaling the lines $a\,o$ and $b\,o$ in Fig. 16 (b), the amount of the oblique forces necessary to create equilibrium, when acting in the direction shown, is found, and $o\,z$ is the thrust exerted or the compression produced in the strut introduced between A and B in (a).

A few explanations at this point may help the student in understanding the meaning of the various lines in force and funicular polygons. When the line $a\,b$, Fig. 16 (b), representing the total load W, has been laid off, and a pole o selected, from which the lines $o\,a$ and $o\,b$ are drawn, the force $a\,b$ has been resolved into the components $a\,o$ and $b\,o$. This construction is based on the triangle of forces, with this difference, that here the resultant $a\,b$ is given and the components found. It is clear that there is a great latitude as to the selection of location of the point o, and therefore of the direction and magnitude of these components.

In a force polygon, any of the components may be resolved into other components, the direction of which may have been given. For instance, in Fig. 16 (b) it is desired to resolve the forces $a\,o$ and $o\,b$ into components, two of which will be

vertical and two parallel with the line AB in the funicular polygon. The line oz, Fig. 16 (b), was drawn parallel with AB and constitutes a component that will be common both to ao and ob; the other component for ao will be az and for ob will be bz. The force triangle, or polygon, aob has thus been divided into two smaller triangles, in one of which the force ao has been resolved into the two components az and oz, while the other force ob has been resolved into the components oz and bz. It is evident that the force oz can be resolved into other components and these again into others; in fact, this process may be carried on indefinitely.

In constructing the funicular polygon in Fig. 16 (a), only the total load $W = ab$ and its components ao and ob were given. The known component of R_1, that is, ao, which is also one of the components of W, was drawn through the point A and intersected the line of action of the force W at O. From this point, a line was drawn parallel with the component ob in the force polygon, and intersecting the line of action of the reaction R_2 at B. The lines AO and OB represent the components of the load W, the directions and magnitudes of which were determined by the selection of the point o in Fig. 16 (b). As the lines of action and magnitude of the other components of the reactions R_1 and R_2 must be the same, they must of necessity be located on the line connecting the points A and B. The line oz in the force polygon parallel with AB, will determine the magnitude of the other components of the reactions R_1 and R_2, az being the vertical reaction R_1, bz the other reaction R_2, and oz the components along the line AB.

To illustrate the locations and actions of the various components more clearly, Fig. 16 (c) has been introduced. In it AA_1 and AO represent the components of the reaction R_1, AO and OB, those of the load W, and OB and BB_1, those of the reaction R_2. In general, it is understood that the lines of action of the components AA_1 and BB_1 are located along the line AB and it is therefore unnecessary to show them separately, as has been done in this case.

The directions in which the various components act are

found from the force polygon in the manner described, but it is necessary to bear in mind that when a certain force in a polygon is to establish equilibrium, its direction conforms to that of the other forces, but if it is to serve as a resultant, it must act in the opposite direction.

Considering the structure in (*a*) as a cord suspending a weight with its ends separated by a compression member in the position of AB, it is evident that the force polygon (*b*) contains the reaction diagram giving the reactions $a\,z$ and $z\,b$ for each end of the equilibrium polygon. In Fig. 16 (*d*), which represents the weighted cord, the reactions R_1 and R_2 coincident in direction with the lines of action of the cord, are the only forces required at the ends of the cord to preserve equilibrium. These reactions are represented in the force polygon by $a\,o$ and $b\,o$, and are equal to the tension in the cord. For the reactions, the two components could be substituted as shown in (*d*) by the dotted lines; without the reactions these forces would create equilibrium. The components of the oblique reaction R_1 in (*d*) are represented in the force polygon by $z\,a$ and $o\,z$, while the corresponding components of R_2 are $b\,z$ and $z\,o$.

Another pole could have been chosen and the force polygon drawn as at (*e*), in which case the equilibrium polygon would have changed correspondingly, assuming the form shown in (*f*). It will be noticed from (*e*) that the vertical components $b\,z$ and $z\,a$ of the oblique reactions always remain the same length and consequently the same amount when $o\,z$ is drawn parallel with $A\,B$ in the diagram (*f*). The direction of the reactions coincident with the direction of the cords and the oblique components of this reaction are alone changed.

21. Assume the conditions of loading shown in Fig. 17 (*a*). The load W of 1,000 pounds is not centrally placed on the beam; consequently, the principle of moments involved in the theory of beams makes the reaction R_1 considerably less than R_2. Lay off in the force polygon, Fig. 17 (*b*), to a scale of $\frac{1}{4}$ inch equals 100 pounds, the line $a\,b$ equal

to 1,000 pounds. Intersecting at any pole *o* draw *a o* and *b o*. From the point *A*, in (*a*), extend a line parallel with *a o* and intersecting the line of action of the weight or the force *W* extended, at the point *O*. From *O*, draw a line parallel with *b o*, Fig. 17 (*b*), intersecting the reaction R_2 at *B*. Connect *A* and *B* as before, and from *o* in (*b*) draw a line parallel with *A B* in the equilibrium polygon and intersecting *a b*. By measuring *b z* and *z a*, the reactions R_2 and R_1 will be found to equal 770 and 230 pounds, respectively.

The student should not be guided by the size of these diagrams, as the size of those employed in practice for solving graphical problems should be considerably larger to give

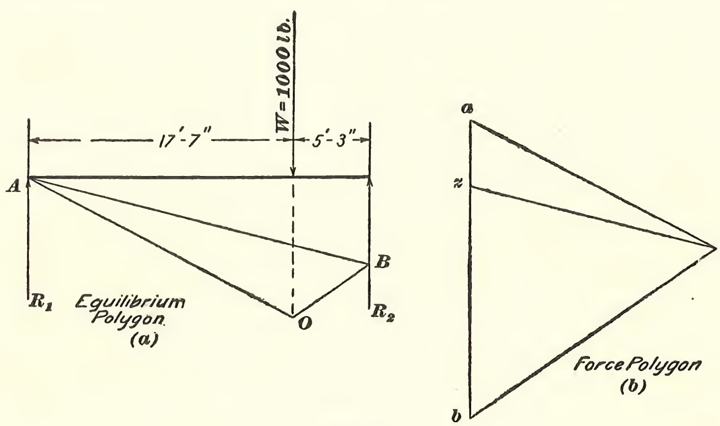


FIG. 17

accurate results. The stress diagram would be laid out to a scale of from 500 pounds to 5,000 pounds to 1 inch, while the scale used for the frame diagram would be from $\frac{1}{16}$ inch to $\frac{1}{4}$ inch to the foot.

22. The application of the force and equilibrium polygons for determining the reactions of non-concurrent forces is not limited to a beam or structure supporting one weight, for they can be used for obtaining the reactions of a simple beam supporting any number of loads acting in any direction. For example, assume that a beam, Fig. 18 (*a*), supports the two loads W_1 and W_2, which are placed at the position fixed

by the given dimensions. In order to determine the reactions R_1 and R_2, lay off the force polygon (*b*) by drawing the load line *a c* and, measuring with some convenient scale, make *a b* and *b c* equal, respectively, to W_1 and W_2. Choose any pole *o* and connect it with the points *a*, *b*, and *c* by radial lines. Draw the equilibrium polygon by commencing at *A* and drawing *A O* parallel with *a o* of the force polygon, intersecting the line of action of the load W_1 at *O*. From this point, draw a line parallel with the line *o b* in the force polygon, intersecting the line of action of the load W_2 at *B*. Finally, draw from *B* a line parallel with *o c* in the force polygon and intersecting the reaction R_2 at *C*. Connect *A* and *C*, as explained in the previous cases, and from *o* in the force polygon draw a line parallel with *A C*, intersecting the load line at *z*; the reactions R_1 and R_2 are found by scaling *z a* and *c z*. If, in the equilibrium polygon, *A O* and *C B* are extended until they intersect at the point O_1, a vertical line drawn upwards from O_1 will divide *A C* into two such parts that the ratio of X to X_1 will be inversely proportional to the ratio of R_1 to R_2; the amount of R_1 and R_2 may be found by designating *A C* equal to the sum of the loads W_1 and W_2, when R_1 will have the same ratio to the total load as X_1 to the whole distance *A C*, and R_2 will bear the same relation to the total load as X to *A C*.

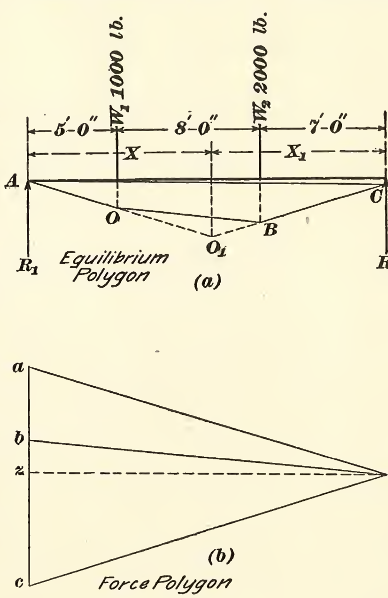


FIG. 18

23. The example shown in Fig. 19 (*a*) is similar to the one just described with the exception that several loads are

applied and there are vertical loads also at each end of the beam. The force polygon, Fig. 19 (*b*), is drawn as in the previous cases, and in drawing the equilibrium polygon the lines of action of the several forces are extended as shown by the dotted lines. In drawing this equilibrium polygon, however, it will be noticed that the line marked *1* is parallel with the second line from the top of the force polygon, or

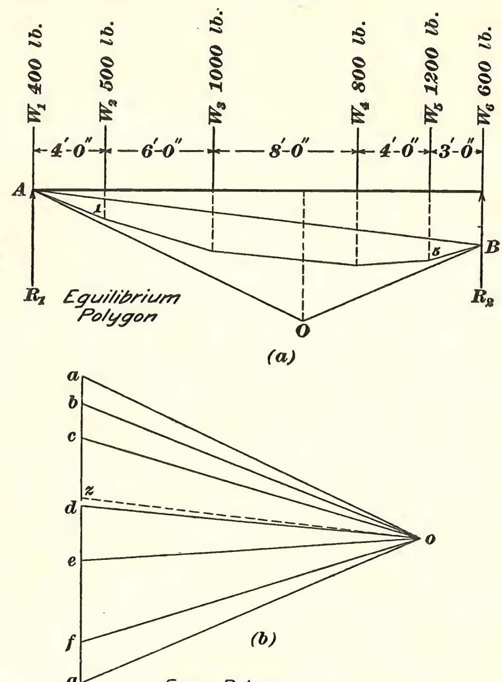
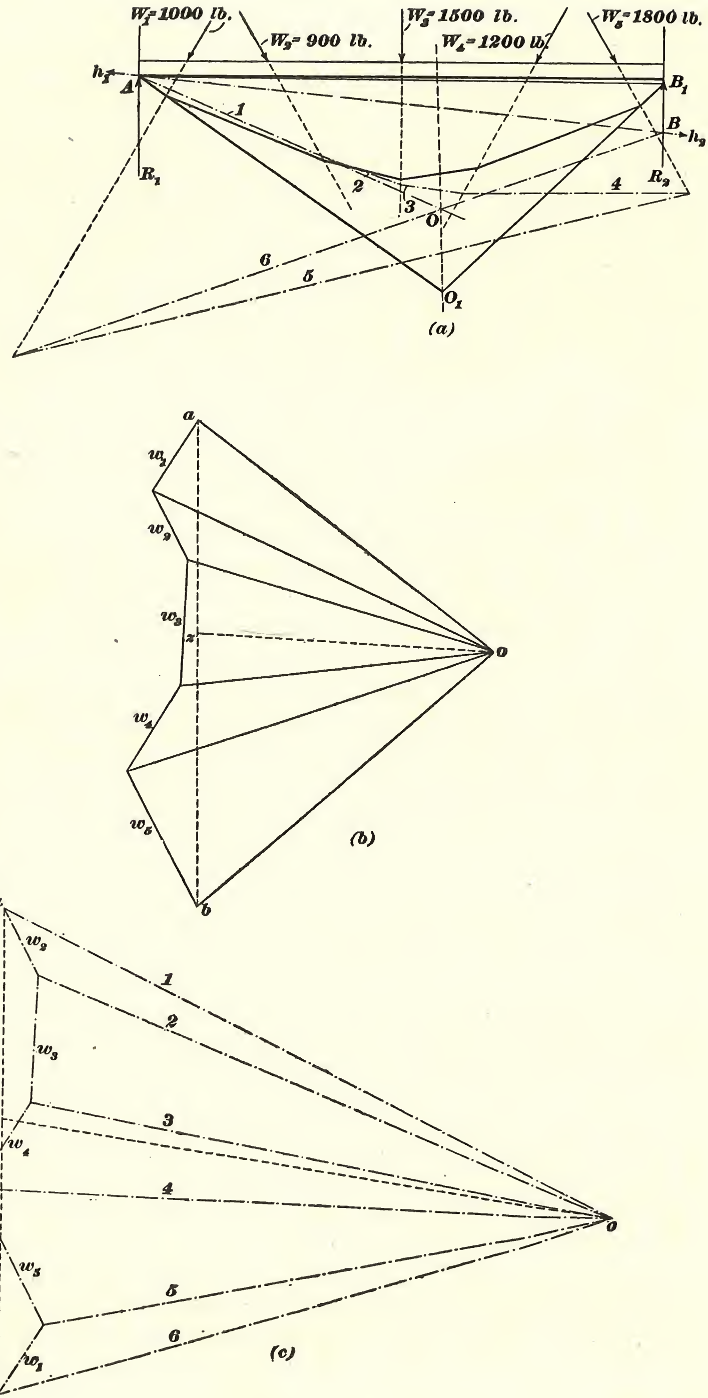


FIG. 19

b o, and that the first line of the force polygon *a o* is represented in the equilibrium polygon by *A O*, while reference to the right-hand end of the figure will show that the line marked *5* in the equilibrium polygon is parallel with the line marked *o* in the force polygon. The reason for this is that the loads W_1 and W_6 are coincident with the reactions R_1 and R_2, respectively. When the load W_1, represented in the force polygon by the force *a b*, is resolved into its two

components *o a* and *o b*, these should, as already explained, be laid off on either side of the line of action of the load W_1, one intersecting the reaction R_1 and the other the line of action of load W_2. In this case the forces W_1 and R_1 being coincident, only the component *o b* can be drawn. The same conditions prevail at the load W_6. While the loads W_1 and W_6, on account of their position, produce no stress in the girder, yet they add their share to the total load and will therefore have to be included among the forces in the force polygon, in this manner affecting the position of the point *z* and, as a consequence, the amount of the vertical reactions R_1 and R_2. If the forces *a b*, *b c*, . . . *f g* were combined, they would equal the total load and would be represented by the line *a g*, the components of which would be *o a* and *o g*. If *A O* and *O B* were drawn parallel with these components they would intersect at the point *O*, which would be located on the line of action of a load, the location and magnitude of which would be such that it would have the same effect as the six loads. The point of intersection *O* of the lines *A O* and *B O* is the center of action of the loads; that is, a vertical force equal and opposed to the loads when located at this point will just balance the loads and create rotary equilibrium, or conversely, it is the position at which a weight, equal to the weights on the beam, suspended from the point *O* on a cord fastened at *A* and *B* will produce the same reactions as the weights on the beam. The reactions at the end of the beam, which are found by measuring *z a* and *g z*, are represented in the equilibrium polygon by R_1 and R_2. The construction of the equilibrium polygon as described usually exists where there are two end loads on the beam or structure, and the student should always bear in mind the fact that the equilibrium polygon is contained between the lines of action of the reactions, and that in the equilibrium polygon there exists the same number of lines as radiate from the pole *o* in the force polygon.

24. The solution of the complicated problem shown in Fig. 20 (*a*) is as readily accomplished as in the simpler



W1=1000 lb.
W2=900 lb.
W3=1500 lb.
W4=1200 lb.
W5=1800 lb.
h1
A
B1
B
h2
R1
R2
1
2
3
4
5
6
O
O1
(a)
a
w1
w2
w3
z
w4
w5
o
b
(b)
a
w2
1
2
w3
z
3
w4
4
o
w5
5
6
w1
b
(c)

FIG. 20

cases previously assumed. The several loads need not be laid off in the force polygon in the regular order, as shown in Fig. 20 (*b*); but in whatever order they are laid off in the force polygon, the same sequence must be observed in laying out the equilibrium polygon. Hence, the load line of the force polygon may be obtained by laying off the loads W_2, W_3, W_4, W_5, and W_1, as shown in Fig. 20 (*c*); and by connecting the points at the ends of the load line, as designated by the line *a b*, the direction of the reactions at the ends of the beam is found. Choose any pole *o* and draw radial lines to the end of each force on the load line. The reactions R_1 and R_2 in the equilibrium polygon will then lie in the direction of the line *a b* in either force polygon, and both equilibrium polygons will be included between these two lines.

Both equilibrium polygons are shown in Fig. 20 (*a*); the one corresponding to the force polygon at (*b*) is shown with solid lines, while the one described from the force polygon (*c*) is shown with dot-and-dash lines. In drawing the latter, commence at the point *A* on the line of action of the reaction R_1 and draw the line marked *1* of the equilibrium polygon parallel with the first radial line in the force polygon. This line in the force polygon connects the intersection of the reaction line *a z* and the force or load w_2 with the pole *o* and should, consequently, be drawn in the equilibrium polygon from the line of action of the reaction R_1 to the line of action of W_2. From this intersection, draw line *2* parallel with the line in the force polygon similarly marked; this extends to the line of action of the force W_3, since line *2* in the force polygon connects the intersection of loads w_2 and w_3 to the pole *o*.

Draw from this intersection, line *3* parallel with the ray *3* in the force polygon, intersecting at W_4 extended. Proceed in this manner around the entire equilibrium polygon until the point *B* has been obtained. From *B* in the equilibrium polygon draw a line to the point *A* and draw from *o* in the force polygon a line parallel to *A B* intersecting the line of the reactions at *z*. The required reactions are measured

from b to z and from z to a, and are equal, by measurement, to R_2 and R_1, respectively.

The system of non-concurrent forces W_1, W_2, W_3, etc. can be held in equilibrium by either one of two systems of reactions: first, by reactions acting in the direction of $B\,O$ and $A\,O$ in the equilibrium polygon and equal in amount to the forces determined by measuring $o\,b$ and $o\,a$ in the force polygon (c); second, by the reactions R_1 and R_2 and the thrusts h_1 and h_2, these forces being the components of $a\,o$ and $o\,b$ in (c).

It is shown in this solution, by comparing the diagrams (b) and (c), that the reactions have the same amount and direction no matter in what order the loads are taken, if the same sequence is followed in drawing the equilibrium polygon. To make this clearer, it may be further stated that in drawing the equilibrium polygon, all that need be observed is that, commencing at the point A, the lines *1*, *2*, *3*, *4*, etc. of the equilibrium polygon are laid off between the lines of action of the several forces and extended in the direction determined by the ray in the force polygon relating to the particular force in the equilibrium polygon from which it is drawn. Thus, the line *1* in the equilibrium polygon, extending between the lines of action of the reaction R_1 and the load W_2 corresponds with line *1* in the force polygon drawn from the intersection of the line $a\,z$, which represents the reaction R_1, and the load line w_2. The line *2* in the equilibrium polygon corresponds with ray *2* in the force polygon and is drawn from the point of intersection of line *1* and the line of action of W_2 extended. Likewise, line *3* in the equilibrium polygon coincides in direction with ray *3* in the force polygon and is drawn from the intersection of line *2* with the line of action of W_3 extended.

The reason for this method of constructing the equilibrium polygon from the force polygon, Fig. 20 (c), is easily seen when it is remembered what was stated in Art. 23 regarding the components of each of the forces on the load line $a\,b$. In each of the force triangles into which the force polygon has been divided, the two rays are components of the force

that constitutes the third side. Lines drawn in the equilibrium polygon parallel with these components should in each case intersect on the line of action of that load or force that, in the force polygon, is represented by the third side of the triangle in which the components are located.

The lines in the equilibrium polygons, which are drawn through the points O and O_1 in the direction of the reactions, give the positions on the lines $A\,B$ and $A\,B_1$, respectively, at which it would be necessary to apply a force equal to the sum of the reactions and acting in their direction, to balance the loads and to produce both rotary and translatory equilibrium. In this instance, these lines are coincident and pass through both points O and O_1.

The results obtained from the two force polygons will be the same, though probably it is always more convenient to take the forces in the order in which they occur. The student should thoroughly familiarize himself with the principles involved in the application of the equilibrium and force polygons; and that he may understand the usefulness of this system of analysis, he should work out for himself by the graphical method the problems called for in the following examples.

EXAMPLES FOR PRACTICE

1. A main girder, having a span of 40 feet, is subjected to the superimposed loads shown in Fig. 21; determine the amount of the reaction at both ends, by the graphical method. Ans. $\begin{cases} R_1 = 7{,}184 \\ R_2 = 5{,}516 \end{cases}$

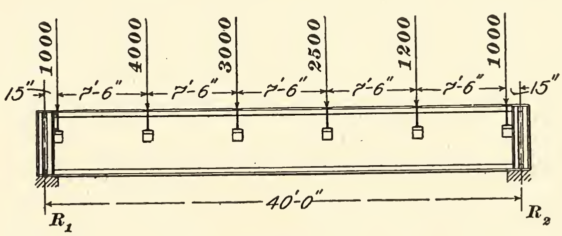


FIG. 21

2. A steel I beam is required to sustain the foot of a slanting column or strut forming one of the supports of a water tank, as

shown in Fig. 22; what will be the direction of the reactions on the walls and their amounts, by the graphical determination, provided the compression in the strut equals 3,000 pounds?

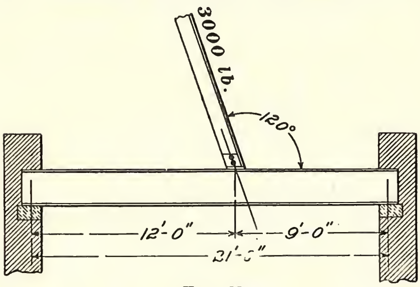


FIG. 22

Ans. Direction: same as load
Amounts: $R_1 = 1,275$
$R_2 = 1,725$

3. A pair of light steel channels in a machine shop is subjected to the pull of several belts and is also required to sustain a load from a hoist and traveler, as shown in Fig. 23; what direction and amounts will the graphical method give the reactions at the column supports?

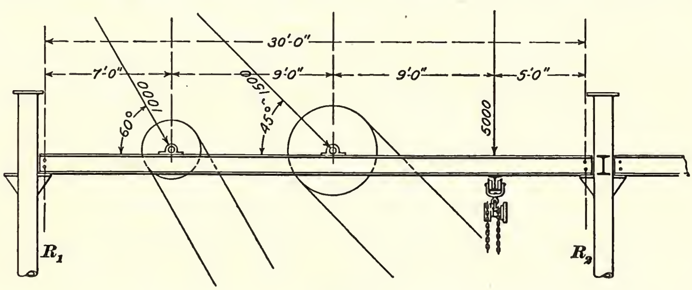


FIG. 23

Ans. Direction: approximately 13° with vertical axis
Amounts: $R_1 = 2,050$
$R_2 = 5,050$

APPLICATION OF GRAPHICAL ANALYSIS

SCOPE OF GRAPHICAL SOLUTIONS

25. The study of the composition of forces has shown that the final effect of two forces can be represented by a third force having a different direction and intensity; and also that any oblique force can be resolved into its vertical and horizontal components. Therefore, when the direction and intensity of one of three forces acting at a joint or connection in a framed structure are given, the other forces can be obtained, provided that the directions or amounts of both, or

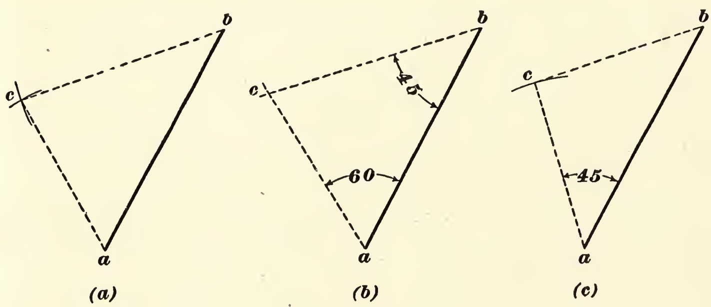


FIG. 24

the direction and amount of one are known; for instance, the forces *a b* in Fig. 24 (*a*), (*b*), and (*c*) are known both as to intensity and direction. It is assumed in the case shown in (*a*) that the amount of each of the two unknown forces is given and that in order to determine these forces it is necessary to know the direction in which they act. To find the unknown directions of the two forces the dividers should be set to scale for one of the forces and an arc struck from *a*, as designated. The dividers being then set for the other

force, an arc is struck from *b* and their intersection at *c* determines the direction of *a c* and *c b*. In (*b*) the directions of the two unknown forces are known either from the fact that they are parallel with another line already determined, or because the angle that they make with *a b* is known. Assuming that the angle that the unknown force extending from *a* makes with the line *a b* is 60°, the line representing this force may be extended indefinitely, as shown. If the angle of the force drawn from *b*, with reference to *a b*, is equal to 45°, it is evident that a line drawn at this angle from *b* will intersect the other force at *c*. Then by scaling *a c* and *b c* the two unknown forces are completely determined, that is, both their direction and intensity are known. In (*c*) the direction and intensity of both *a b* and *a c* are known; therefore, the point *c* can be located by scale and *b c* drawn. Consequently, both the direction and amount of *b c* may be obtained in this manner.

26. From the foregoing it has been determined that the following statements are true of any system of three forces:

1. That the unknown direction of two forces can be obtained when their amounts and the amount and direction of the third force are known.
2. That the amount of two unknown forces may be found when their direction and the amount and direction of the third force are known.
3. That the direction and amount of a force may be obtained when the direction and the amounts of the other two forces are known.

In the application of graphical statics, the truth conveyed by the first statement is seldom employed, but the second and third principles form the basis of the science of graphical statics as applied in the solution of stresses in framed structures.

In Fig. 25 (*a*) is shown a diagrammatical figure that represents a panel point, or joint, on the rafter member of a roof truss. The known stresses about this point are the

compression in the lower portion of the rafter member and the vertical force W that represents the load at the panel point applied through the purlin that is secured to the frame at this position. These two known forces are shown by heavy lines, while the unknown stresses exist in the upper portion of the rafter member and the strut, the latter being designated by light lines.

Assume that the weight W is equal to 1,000 pounds and that the stress in the lower portion of the rafter member is 6,000 pounds. The problem is to determine the stress in the upper portion of the rafter member and in the strut. In (b), lay off on a line parallel with the rafter member, a distance equal, by scale, to 6,000 pounds. It is known that the direction of the stress in this member is upwards

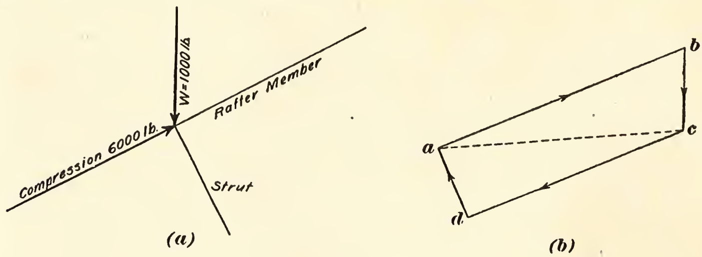


Fig. 25

toward the joint, so that the line just drawn extends from a to b in the direction of the arrow, as shown. From b, the line bc is laid off equal, by the same scale, to the amount of W or 1,000 pounds, and downwards. The resultant of these two forces will extend, as shown by the dotted line, from a to c. Let this resultant, therefore, represent the base line on which the two unknown forces are to be constructed; that is, let it represent the line ab shown in Fig. 24 (a), (b), and (c). The length and amount of this resultant ac is known.

By applying, therefore, the second principle stated above, the amount of the two unknown forces may be found. In order to determine these forces, draw from c a line parallel with the rafter member and from a a line parallel with the strut. Then,

by measuring with the same scale to which *a b* and *b c* were laid off, the stress in the strut can be found from *a d*, while the stress in the upper portion of the rafter member is found by measuring *c d*. The polygon of forces around the entire joint extends from *a* to *b*, from *b* to *c*, from *c* to *d*, and from *d* to *a*; since the polygon closes, the joint is in equilibrium of translation, and because the forces are concurrent, the joint has no rotary tendency. Though it was not necessary to draw the resultant *a c*, this was done to show that every force polygon is made up of a series of triangles of forces, and the resultant of any number of forces of the system is obtained by a line connecting the end of any force with the point of commencement of the polygon. For instance, the resultant of *a b* and *b c* is *a c*, while the resultant of *a b*, *b c*, and *c d* is *d a*; in the same manner the resultant of *b c*, *c d*, and *d a* is *a b*.

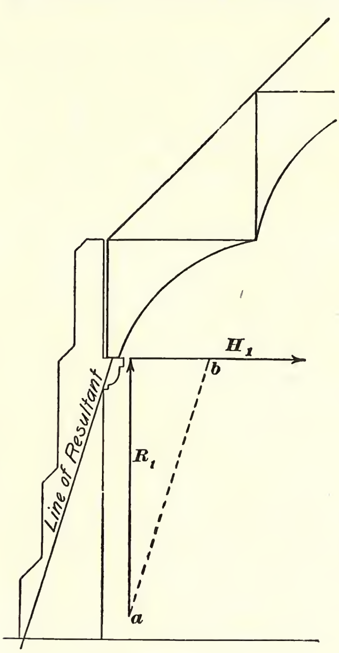


Fig. 26

The application of the third principle is usually employed where it is necessary to determine one of the unknown forces by means of calculation, as will be further explained; it is also used in drawing the reaction diagrams. In explanation of the latter, assume that in order to hold the foot of the roof truss designated diagrammatically in Fig. 26, there necessarily exists a vertical force, as shown by R_1, and a horizontal thrust, as designated by H_1. It is desired to determine the resultant of these two forces, in order to find at what position on the base line this force will intersect, so that it may be decided whether the abutment is in equilibrium. The amount of the thrust H_1 is laid off to scale, as shown. The reaction

R_1 is known and the points a and b are consequently located. The resultant will therefore extend from a to b, and when drawn from the center of the foot of the truss, the effect of these two forces on the abutment is determined and exists as shown.

DIAGRAMS

27. In the application of graphical statics to the determination of stresses in roof trusses and framed structures, the direction of all internal and external forces, excepting the reactions, are known, and the solution resolves itself into the determination of the amounts of the forces only. In order, therefore, that the direction of the several external and internal forces that are known may be applied in drawing the diagram for obtaining the stresses, it is necessary that an accurate outline of the truss or framed structure shall be laid out to scale. In this diagram, which is drawn according to the principal dimensions, the exact direction of all the external forces and the internal members, that represent the internal forces, are accurately designated. Such a diagram, which is practically the preliminary sketch or study of the structure, is called the **frame diagram,** from the fact that it represents the framework, or skeleton, of the structure. The diagram that is drawn in order to determine the stresses and in which each line represents graphically, in its relation with the other lines of the diagram, the several forces exerted on the structure, either internally or externally, is called the **stress diagram,** though sometimes erroneously termed *strain diagram.*

The frame diagram for a simple roof truss is shown in Fig. 27 (a), while the stress diagram is shown in (b) of the same figure. The frame diagram is drawn to a convenient scale of a certain number of inches to the foot, but since lines in the stress diagram represent forces in direction and intensity, the stress diagram is always drawn to a convenient scale of pounds equal to some unit of linear measurement, such as 1 inch; for instance, the frame diagram shown in the figure may be drawn to a scale of $\frac{1}{4}$ inch = 1 foot, while the

stress diagram may be laid out to such a scale that every inch in the length of the line represents 1,000 pounds, or if a tenth scale is used each $\frac{1}{10}$ inch equals 100 pounds. The lines in the frame diagram representing the forces are never drawn to scale, as they simply show the direction of the forces. Sometimes, however, it is customary to lay out a stress diagram upon or connected with the frame diagram in order to obtain, possibly, the vertical or horizontal components of an oblique force, or to find the resultant reaction for a system of forces representing the independent reactions due to different external forces acting on the structure; for example, in Fig. 28 is shown, in light lines, a stress diagram drawn in conjunction with the frame diagram. Here it was deemed expedient to determine one force, such as R_2, that would equal in effect the horizontal force H_2, the vertical force V_2, and the oblique force O_2. The direction and amount of the resultant R_2 was found by laying off, to scale, the amounts of V_2, H_2, and O_2 on their respective coincident or parallel lines, *ab*, *bc*, and *cd*, the resultant R_2 being equal in intensity and direction to *da*. When this force R_2 is introduced instead of the three forces, the equilibrium of the structure is still maintained.

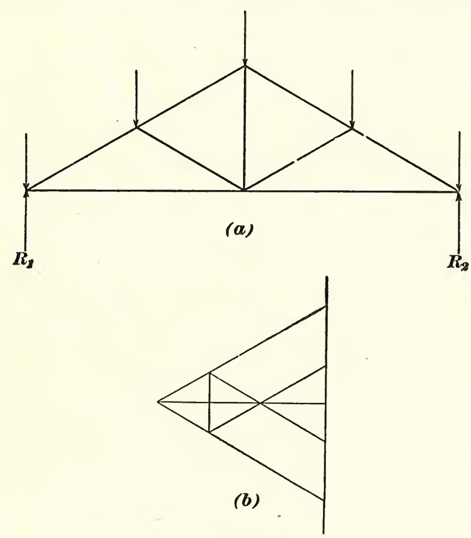


FIG. 27

In drawing the frame diagram, any usual scale, as $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{3}{4}$, or $\frac{1}{2}$ inch to the foot, can be used, though probably the $\frac{1}{8}$- and $\frac{1}{4}$-inch scale will be found most convenient. In laying out stress diagrams, the tenth scale can be more

readily employed than the usual inch scale divided into 8ths and 16ths.

28. The frame and stress diagrams are often called *reciprocal diagrams*, from the fact that for every system of forces in the frame diagram there exists in the stress diagram a series of lines that are respectively parallel with the lines of the forces in the frame diagram; and for every joint at which a system of concurrent forces exists in the frame diagram, there is a corresponding polygon of forces in the stress diagram. In determining the stresses on any framed structure, it is usually necessary to draw several stress

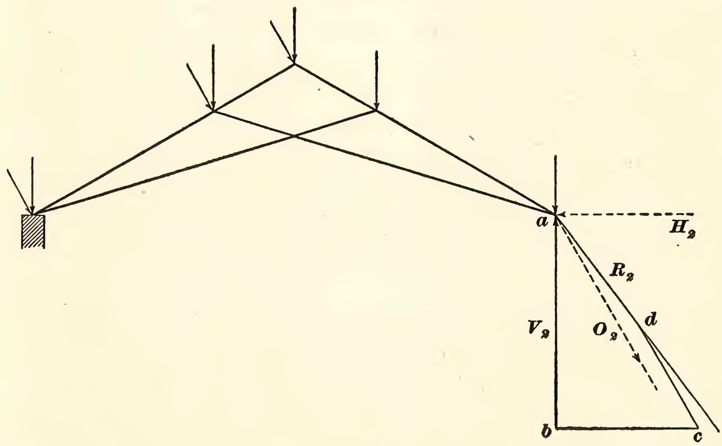


FIG. 28

diagrams; for instance, to find the stresses existing in the truss, it is customary to draw at least a vertical-load diagram and a wind-load diagram, and sometimes it is advisable to draw also a snow-load diagram and even a stress diagram for the wind load on one side of the roof and the snow load on the other. For each stress diagram, the frame diagram remains the same, with the exception that the direction and amounts of the loads at the panel points, as well as the reactions, change. Oftentimes, one outline of the framed structure embodies all of the forces acting on the truss, though where a number of stress diagrams are to be drawn,

considerable confusion is usually avoided by drawing separate frame diagrams. In the design of all roof trusses, the wind and vertical loads should be considered separately, though the less conservative engineers do not consider the separate effects of the wind and snow on spans of less than 100 feet.

NOTATION

29. In order that the external forces or the internal stresses exerted in the frame diagram may be intelligently designated in the stress diagram, so that by inspection one may know that a particular line in the stress diagram designates the amount and intensity of the stress in a particular member in the frame diagram, some convenient **notation** must be employed. A notation should be used such that, in analyzing the stresses around a single joint, the polygon of forces may be readily traced and the completion of the polygon of forces will be immediately known when the point from which the polygon was started has been finally reached. The system of lettering, or notation, commonly used in graphical statics, consists in placing a capital letter in every space throughout the frame diagram between the several members and the forces acting externally. The lettering is usually commenced at the left-hand end of the diagram and runs around the figure in the direction traveled by the hands of a clock; the internal spaces are then lettered in alphabetical order, commencing likewise at the left-hand end of the figure. The middle point of the diagram, or that space included between the two principal reactions, is usually denominated by the letter Z.

In Fig. 29 (a) is shown a frame diagram lettered with the notation just described. The first vertical load at the left-hand end of the truss is known as the load, or force, AB; the second load as BC, etc. The three divisions of the tie-member beginning at the left are designated as GZ, IZ, and KZ, while the several portions of the rafter members are known as GB, CH, JD, and EK. The struts are GH and JK, while the tension rods are denominated HI and IJ.

The stress diagram that would be laid out in order to analyze the stresses in the frame diagram at (*a*) is shown at (*b*); the same letters designate the stresses or forces similarly marked in the frame diagram, with the exception that small, or lower-case, letters are used instead of capitals; for instance, *a b* represents to scale the force *A B*; *b c* represents the direction and intensity of the force *B C*; *c d*

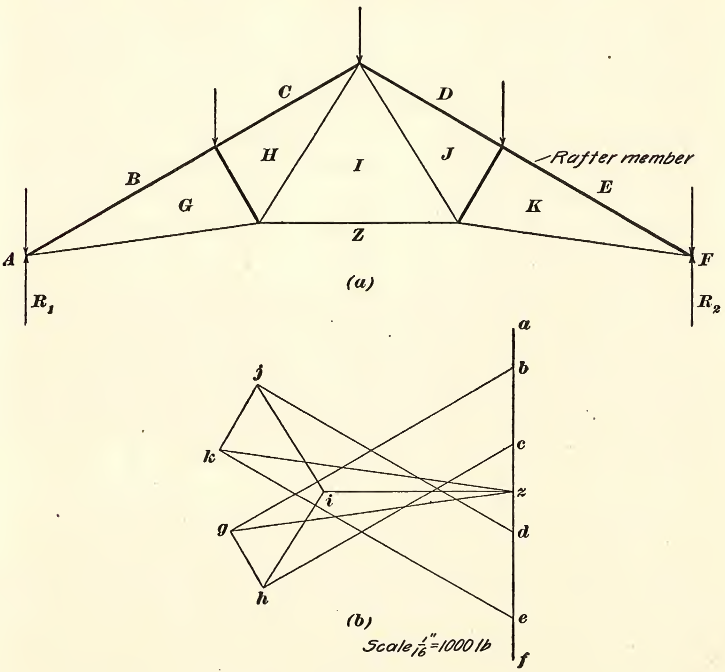


FIG. 29

likewise stands for *C D* in the frame diagram. The reaction R_1 is designated in the stress diagram by *z a*, while *f z* will give, on measuring, the amount of the reaction R_2.

The convenience of this notation consists in the fact that the frame and stress diagrams bear a peculiar relation to each other in that where any system of forces in the frame diagram acts at a single point, there is a corresponding closed polygon represented in the stress diagram. For

example, the apex of the truss is a point at which five forces meet—they are the vertical load CD, the stress in the right-hand rafter member DJ, the stress in the oblique tension member JI, and the stresses in the left-hand rafter member and tension member, respectively, designated as HC and IH. In the stress diagram these forces do not meet at a single point, but form the polygon $cdjih$, the last line hc of the polygon making a closed figure. On the other hand, any closed figure in the frame diagram, such as the triangular space including the letter G, is designated in the stress diagram by a number of lines meeting at a common point; that is, the space G in the frame diagram is enclosed by the members of the frame designated as BG, GZ, and GH, while in the stress diagram the lines that represent the stresses in these several members meet at the point g, and are bg, gz, and gh.

It is not absolutely necessary to use this system of lettering, for any letters or numbers may be employed, so long as no two spaces in the truss are marked alike. It is well, however, to have some definite system in engineering work, and the foregoing, being as convenient as any that can be suggested, is universally adopted.

DETERMINATION OF EXTERNAL FORCES

30. Before the stress diagram for any framed structure can be drawn, it is necessary to complete the polygon of external forces, which consists of the loads on the frame and their reactions. The loads supported on a frame are always considered as concentrated at the joints of the frame; in designing roof trusses, the purlins are either located at the joints along the rafter members or else the loads are considered as being transmitted to these points by the transverse strength of the member between the panel points. In Fig. 30 is shown a simple roof truss with its loads and reactions. The panel points, at which the loads on the rafter members are considered as being concentrated, are a, b, c, d, and e. The load at a is the portion of the roof

supported by one-half of the portion of the rafter member between *a* and *b*, while the load on the joint *b* would be that supported by one-half of the rafter member, or one-half of *a b* and *b c*. The load at the apex of the roof is one-half that portion of the truss included between *b c* and *c d*.

The principal difficulty in determining the polygon of external forces is to find the amount of the reactions, or the forces opposed to the loads, in order to create equilibrium in the structure. When the frame or truss is symmetrical and the line of action of the reactions is parallel with

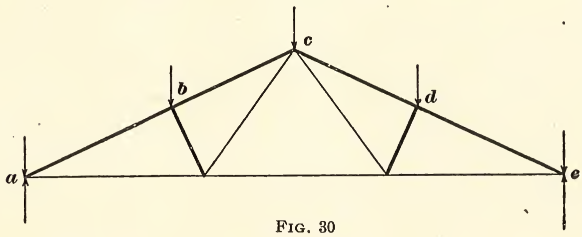


Fig. 30

the direction of the loads, the reactions are obtained by inspection, for they are each equal to one-half the sum of the loads on the truss. Should the truss, however, be unsymmetrically loaded or should the reactions be other than parallel with the line of action of the loads, their direction and their amounts may be calculated or found by the graphical method.

In finding the reactions of a truss it will be noted that when all the loads are vertical, both reactions will be vertical. When the load is applied at a certain angle, the two reactions will be parallel to each other and also parallel to the direction of the load. When, however, both the vertical load, as the dead load, and the oblique load, as the wind load, act together, the reactions may not be parallel to each other or to any of the loads applied. If one end of the truss is on a roller, the reaction at that end will be vertical, and the other may be easily found; but if both ends of the truss are fastened to the building, the directions of the reactions will be more difficult to obtain. In such a case it is better to find first

the reactions for the vertical loads alone, and then the reactions for the oblique loads at the same angle, and then combine these reactions for the total reaction.

DETERMINING THE REACTIONS BY CALCULATION

31. If the frame be regarded as a solid body acted on by external forces, little difficulty will be experienced in determining the method of procedure for finding the reactions.

In Fig. 31 (*a*), (*b*), and (*c*) is shown a type of simple roof truss loaded in several different ways. In (*a*) is shown a truss unsymmetrically loaded, so that the reactions will not each be equal to one-half of the total load on the truss, though from the fact that the reactions coincide with the direction of the loads, it is known that the sum of the reactions is equal to the sum of the loads.

In order to determine the reaction R_2, the truss is considered as being hinged at the point c and the algebraic sum of the moments of all the external forces about this point should equal zero, or the sum of the moments of the loads equals the moment of the reaction R_2, the latter moment being the product of R_2 and the length S. If the sum of the moments due to the loads is divided by the lever arm of R_2, the amount of R_2 will be determined. If the loads on this truss are represented by w_1, w_2, w_3, etc., and their respective lever arms about the point c designated by x_2, x_3, etc., the algebraic sum of the moments about the point c will equal $w_2 x_2 + w_3 x_3$, etc.; and since the lever arm of R_2 is equal to the span of the truss, or S, the amount of R_2 may be determined by the expression

$$R_2 = \frac{w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5}{S} \qquad (1)$$

As the load w_1 is coincident with the reaction R_1, its moment is zero and is therefore not included in the formula.

For example, assume that the span of the truss is equal to 40 feet and that the distances x_2, x_3, x_4, x_5 are equal, respectively, to 10, 20, 30, and 40 feet; w_1 equals 1,000 pounds; w_2

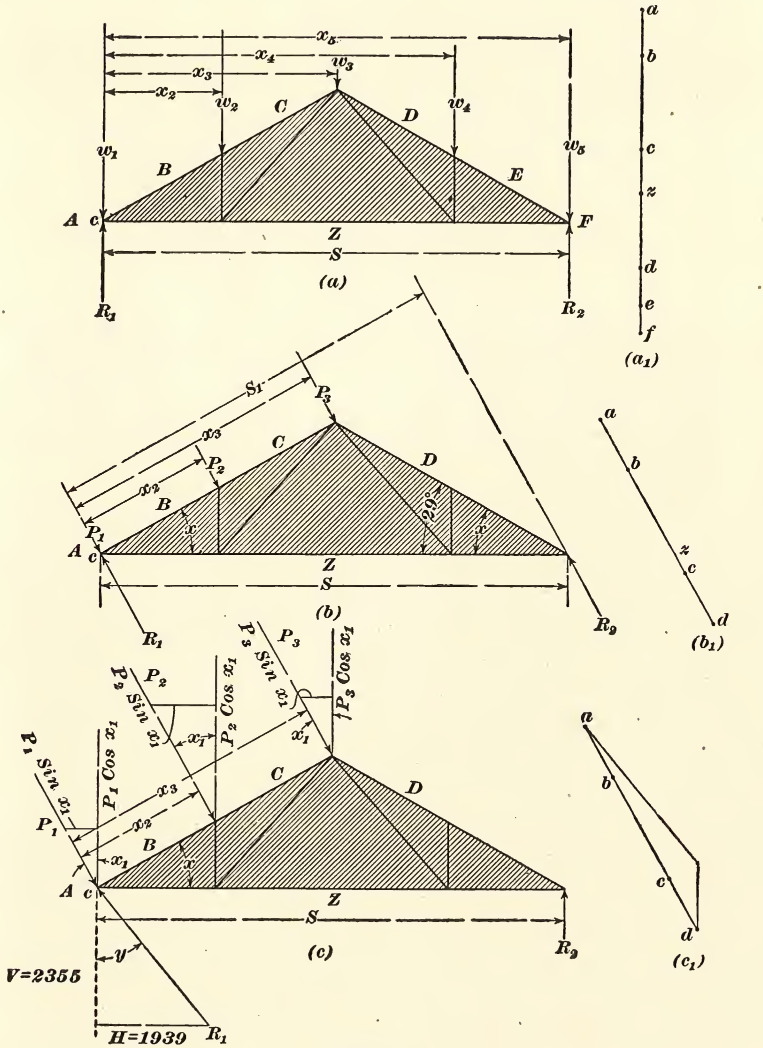


FIG. 31

equals 2,000 pounds; w_3 equals 2,500 pounds; w_4 equals 800 pounds; w_5 equals 600 pounds.

Since the calculation for moments can best be systematically arranged by tabulation, the sum of the moments of the loads will be expressed as follows:

$$\begin{aligned}
w_2 x_2 &= 2{,}000 \text{ lb.} \times 10 \text{ ft.} = 20000 \text{ ft.-lb.}\\
w_3 x_3 &= 2{,}500 \text{ lb.} \times 20 \text{ ft.} = 50000 \text{ ft.-lb.}\\
w_4 x_4 &= 800 \text{ lb.} \times 30 \text{ ft.} = 24000 \text{ ft.-lb.}\\
w_5 x_5 &= 600 \text{ lb.} \times 40 \text{ ft.} = \underline{24000} \text{ ft.-lb.}\\
\text{Total moment about } c &= 118000 \text{ ft.-lb.}
\end{aligned}$$

The moment of R_2 about the point c must equal the sum of the moments of the loads, and hence the reaction R_2 equals $118{,}000 \div 40$, or 2,950 pounds. The sum of the reactions must equal the sum of the loads, and since the sum of the loads equals 6,900, the amount of R_1 will equal $6{,}900 - 2{,}950$, or 3,950 pounds.

The polygon of external forces at (a_1) may now be laid out, and because all the external forces on the truss are coincident in direction, the force polygon will extend in a straight line. This polygon will be laid out from a to b, from b to c, from c to d, from d to e, from e to f, and back from f, locating the point z, by scale, at a distance from f equal to the reaction R_2, or 2,950 pounds. If the polygon of the external forces has been accurately laid out, the distance $z a$ will, on scaling, be found to equal the reaction R_1.

32. In Fig. 31 (b) is shown the frame diagram of a roof truss sustaining on the left-hand rafter member, at the several panel points, the wind loads P_1, P_2, and P_3. It is considered that the roof truss is securely fixed to a wall at both ends, and under such conditions the reactions coincide with the direction of the normal wind loads. Since this is similar to the preceding case, the sum of the reactions is equal to the sum of the loads, but the reactions, by inspection, are evidently not equal to each other.

The amount of R_2 may be determined by considering the moments of the external forces with the point c as the center of moments. The lever arms of the loads P_2, P_3 are at right

angles with the line of action of the wind pressure, which is normal to the slope. They are therefore parallel with the rafter member or slope. The reaction R_2 exerts its force through a lever arm at right angles to its action, as represented by S_1, and this distance may be obtained by extending the line of action of R_2 and scaling, or it may be calculated. The distance S_1 is always equal to the span of the truss multiplied by the cosine of the angle x; or expressed algebraically, where S = the span of the truss, $S_1 = S \cos x$.

The equation for determining the reaction R_2 of the roof truss shown in (b) may then be expressed by the formula

$$R_2 = \frac{P_2 x_2 + P_3 x_3}{S_1}$$

which is the same as

$$R_2 = \frac{P_2 x_2 + P_3 x_3}{S \cos x} \qquad (2)$$

To apply this information it will be assumed that it is desirable to obtain the amount of R_2 when $P_2 = 2{,}000$ and $P_3 = 1{,}000$ pounds, the distances x_2 and x_3 being equal, respectively, to 11.43 and 22.86 feet. The angle x is, approximately, 29°; then S_1 equals the cosine of 29°, or .87462 multiplied by the span, or 40 feet, which gives 34.98 feet. When this value has been obtained, the calculation for the reaction R_2 will be as follows:

Positive moments of wind pressure about the point c are:

$$P_2 x_2 = 2{,}000 \text{ lb.} \times 11.43 \text{ ft.} = 22860 \text{ ft.-lb.}$$
$$P_3 x_3 = 1{,}000 \text{ lb.} \times 22.86 \text{ ft.} = \underline{22860} \text{ ft.-lb.}$$

Total moment due to wind pressure = 45720 ft.-lb.

This moment, in order that the frame may be held in equilibrium, must be equal to the moment of R_2 about the point c, and by dividing the amount just obtained by S_1, R_2 is found to be equal to 45,720 ÷ 34.98, or 1,307 pounds. The amount of R_1 is found by deducting the amount of R_2 from the sum of the loads; therefore, if P_1 equals 1,000 pounds, R_1 equals 4,000 − 1,307, or 2,693 pounds.

The stress diagram, or polygon of external forces, for the frame shown in (b) is designated in (b_1). The load line in this

instance is oblique, since it must coincide with the direction of the external forces, and when the forces *a b*, *b c*, and *c d* have been laid off and the amount of the calculated reaction R_2 measured from *d*, thus locating the point *z*, the length measured from *z* to *a*, by scale, will check the calculations, if it equals R_1. Having in this manner located the point *z*, the polygon of external forces will extend from *a* to *b*, from *b* to *c*, from *c* to *d*, from *d* back to *z*, and from *z* to the starting point, thus completing the figure.

If both the loads shown at (*a*) and (*b*) were acting simultaneously, as they often are, the reaction at the left-hand end of the truss would be the resultant of the two reactions R_1 already found, and the reaction at the right-hand end of the truss would be the resultant of the two reactions R_2 already found.

33. The frame shown in Fig. 31 (*c*) is loaded with several oblique panel loads caused by the wind pressure; the moments of these loads about the point *c* are determined in the same manner as were the moments of the loads shown in (*b*). In this instance, however, the truss is considered as being supported on a roller bearing at the right-hand end in order to allow lateral play for the contraction and expansion of the metallic frame. This bearing is regarded as frictionless, so that there is no horizontal resistance whatever, in consequence of which condition the reaction R_2 under a roller bearing is always considered as vertical in direction. Hence, the problem resolves itself into the determination of the amount of this reaction R_2. The sum of the moments due to the several wind loads on the rafter members is the same as for the frame in (*b*) and is equal to $P_2 x_2 + P_3 x_3$. The leverage of R_2, since it acts vertically about the point *c*, is equal to the span of the truss, so that the amount of the reaction R_2 is determined from the formula

$$R_2 = \frac{P_2 x_2 + P_3 x_3}{S} \qquad (3)$$

For example, assume that P_2 and P_3 equal, respectively, 2,000 and 1,000 pounds, while x_2 and x_3 equal 11.43 and

22.86 feet; the span, as before, being equal to 40 feet, then

$$R_2 = \frac{2{,}000 \times 11.43 + 1{,}000 \times 22.86}{40} = 1{,}143 \text{ pounds}$$

The other reaction, or R_1, is not vertical, because the truss is fixed at the end, and in consequence the actual sum of the reactions is not equal to the sum of the loads.

It is known, however, that for any frame to be in equilibrium, the algebraic sum of the vertical and horizontal components of all the forces acting on that frame must equal zero. The reaction R_2 is vertical, and consequently has no horizontal component. It is evident, therefore, that the reaction R_1 must have a horizontal component equal to the horizontal components of all the loads, in order that the algebraic sum of the horizontal components of all the loads acting on the truss may be equal to zero. Each of the wind loads P_1, P_2, and P_3 may be regarded as one side of a triangle in which its components constitute the remaining sides. Knowing that the angle x_1 is equal to the angle x, the value of either of the components may be found by means of one of the trigonometric functions. Thus, the horizontal component is equal to $P \sin x_1$, and the sum of the horizontal components of the three oblique forces on the rafter member will equal $(P_1 + P_2 + P_3) \sin x_1$. The sine of 29° is equal to .48481, and the sum of the loads P_1, P_2, and P_3 is equal to 4,000 pounds, so that the sum of the horizontal components of the oblique forces, or H, equals 4,000 $\times$ .48481, or 1,939 pounds.

Having in this manner obtained the horizontal component that R_1 must possess in order to equal the horizontal components of the loads, it is necessary in a similar manner to determine the vertical component of R_1. Since R_2 acts vertically, its amount must be subtracted from the sum of the vertical components of the oblique forces P_1, P_2, and P_3. Trigonometrically, the sum of the vertical components of the pressures equals $(P_1 + P_2 + P_3) \cos x_1$, or substituting the values, the vertical component $V = 4{,}000 \times .87462$, or 3,498 pounds. From this must be deducted the entire amount of R_2, and when this is done there remains for a

vertical component of R_1 an amount equal to $3{,}498 - 1{,}143 = 2{,}355$ pounds.

It is now evident that since both the vertical and horizontal components of R_1 have been obtained, the amount and direction of R_1 may be found.

The amount of R_1 may be found by considering the vertical and horizontal components as the two sides of a right triangle and calculating the hypotenuse by the rule of Pythagoras, that is, by taking the square root of the sum of the squares of the vertical and horizontal components. Employing this method, $R_1 = \sqrt{2{,}355^2 + 1{,}939^2}$, or 3,050. The direction of this reaction is still to be determined. The angle y that it makes with the vertical may be found by either of the following trigonometric functions: secant $y = \frac{R_1}{V} = \frac{3{,}050}{2{,}355}$, or cosine $y = \frac{V}{R_1} = \frac{2{,}355}{3{,}050}$. In the absence of a secant table, the latter formula is used. It will be found from the above that secant y equals 1.295, and cosine y equals .7721, either of which corresponds to an angle of about 39° 30′; therefore, the reaction R_1 will be of the intensity calculated, or 3,050 pounds, and will extend in a direction to the right of the vertical at an angle of 39° 30′.

The calculations for R_1 need not have been made, for both its intensity and direction could be obtained graphically in laying out the polygon of external forces. Fig. 31 (c_1) shows the polygon of external forces for the frame shown in (c). The load line extends from a to b, from b to c, from c to d, and then, since the reaction R_2 is the vertical distance from d to the point z, the distance from d to z is made equal to the amount of the reaction R_2, or 1,143 pounds. When the point z has been located, the amount of the reaction R_1 and its direction may be determined by a line connecting the points z and a. If this line in the particular example that has been considered measures, to scale, 3,050 pounds and extends in a direction of 39° 30′ with the vertical, it will coincide with the values obtained by the calculation and will check the diagram. The polygon of external forces in this

case will then extend from *a* to *b*, from *b* to *c*, from *c* to *d*, from *d* vertically to the point *z*, and from *z* back to the starting point of the polygon or *a*.

GRAPHICAL METHOD OF OBTAINING THE REACTIONS

34. The reaction of a system of forces acting on a framed structure may be entirely determined by the graphical method. It is assumed that the roof truss shown in Fig. 32 (*a*) is loaded with the vertical loads *A B*, *B C*, etc. at each panel point and that it is desired to obtain the

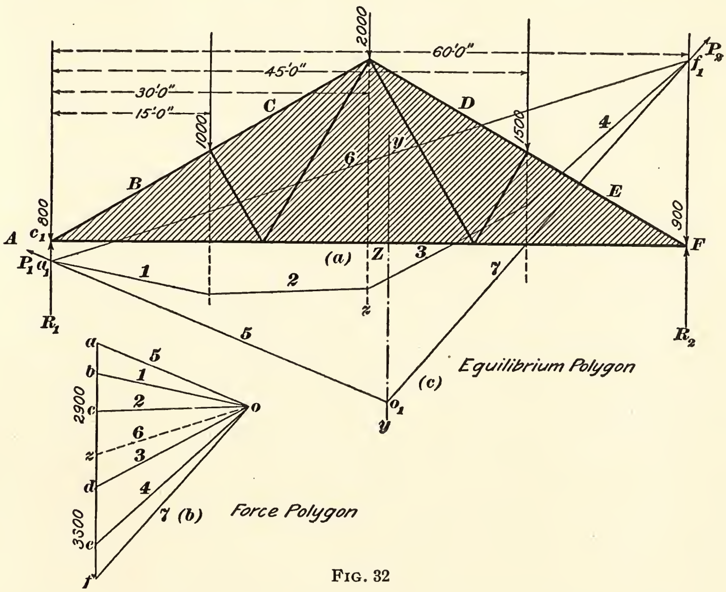


FIG. 32

amount of the reactions R_1 and R_2. If the loads had been symmetrical in amount on each side of the truss, these reactions would have been equal and would have corresponded in amount with one-half the load on the truss. It will be noted, however, that the right-hand half of the truss is more heavily loaded than the left-hand portion, and therefore the reactions will not be equal. Before the stress diagram can be drawn, the unknown external forces R_1 and R_2 must be obtained in

order that the location of the point z in the stress diagram may be determined. These reactions can be calculated by the principles of moments explained in Art. **31**, or they can be determined by the graphical method. This method consists in drawing the force polygon (b), and from this constructing the equilibrium, or funicular, polygon (c).

In laying out the force polygon, lay off the load line designated from a to f. While it is more systematic to lay off the loads on this line in the order in which they occur around the truss, it is not necessary that this procedure should be followed, for these loads may be laid off in any convenient succession. Each portion, however, of the load line must truly represent, to scale, the amount of the force that it designates and its direction. Any point or pole o is selected and lines are drawn from it to the several divisions on the load line. The figure will then appear as shown, with the exception that the line $o\,z$ has not as yet been determined, for the direction of this line is what is required in order to find the amount of the reactions R_1 and R_2. Having completed the force polygon (b), proceed with the equilibrium polygon (c). In drawing this polygon, first extend the line of action of all the forces acting on the frame, as shown by the dotted lines, and commence the diagram at the left-hand end by drawing the line *1* parallel with $b\,o$ in the force polygon and from the intersection of the line *1* with the line of action of the force $B\,C$ draw line *2* parallel with $c\,o$ in the force polygon; and where this line intersects the line of action of CD draw the line *3* parallel with $d\,o$. Likewise, from the intersection of *3* with the line of action of DE, draw *4* parallel with $e\,o$ in the force polygon. This latter line will intersect the line of action of R_2, and from this intersection f_1 draw $f_1\,o_1$ in the same direction as $o\,f$ in the force polygon, while from a_1, the left-hand end of the equilibrium polygon, extend the line $a_1\,o_1$ parallel with $a\,o$ in the force polygon. Connect the points f_1 and a_1 in the equilibrium polygon and draw from o in the force polygon a line parallel with $f_1\,a_1$. The intersection of this line $o\,z$, or *6* in the force polygon, will divide the vertical load line into

fz and za, which, to the scale to which the force polygon was laid out, represent the reactions R_2 and R_1, respectively.

In the equilibrium polygon, the lines *1*, *2*, *3*, and *4* represent the form that a cord or string would assume if it supported the loads BC, CD, and DE at the several points where it changes direction, and its ends were attached at a_1 and f_1. In order that the system might be in equilibrium, the cord would require either the oblique reactions marked P_1 and P_2, or a compression member exerting a compressive stress as represented by the line $a_1 f_1$, and the vertical reactions R_1 and R_2. It is evident from this that P_1 and P_2, or their equivalents, the forces $a_1 o_1$ and $f_1 o_1$ in the equilibrium polygon, are the resultants of two sets of components R_1 and $f_1 a_1$, and R_2 and $a_1 f_1$, respectively. A force designated by two letters should be read in the direction in which the force acts; that is, the letter toward which the arrow representing the direction of the force points should be read last. In this instance, $a_1 f_1$ represents a compressive stress and when speaking with reference to the point a_1, the force is designated as $f_1 a_1$, while if the point f_1 is being considered, the force should be read $a_1 f_1$. In the force polygon, ao corresponds with the force P_1 in the equilibrium polygon and of, with P_2. The components of ao in the force polygon are az and zo, while the components of the force fo are fz and zo. By measuring fz to the scale to which the load line was laid out, it will be found to equal, approximately, 3,300 pounds, while za, measured with the same scale, equals 2,900 pounds.

To prove, or check, the diagrams, the moments of the several external forces may be calculated about the point of application of either R_1 or R_2. Considering the loads on the roof truss as giving positive moments about c_1, their sum is obtained by the following calculation:

Moment of AB =	800 lb. × 0 ft. =	00000	ft.-lb.
Moment of BC =	1,000 lb. × 15 ft. =	15000	ft.-lb.
Moment of CD =	2,000 lb. × 30 ft. =	60000	ft.-lb.
Moment of DE =	1,500 lb. × 45 ft. =	67500	ft.-lb.
Moment of EF =	900 lb. × 60 ft. =	54000	ft.-lb.
	The sum of moments =	196500	ft.-lb.

The leverage of R_2 about c_1 is 60 feet, and R_2 is found by dividing the sum of the positive moments by this distance; by the calculation it equals 3,275 pounds, while the value of R_1 is the difference between this amount and the sum of the loads, or 2,925 pounds, which corresponds, approximately, with the values obtained by the diagram, thus proving the correctness of the graphical solution.

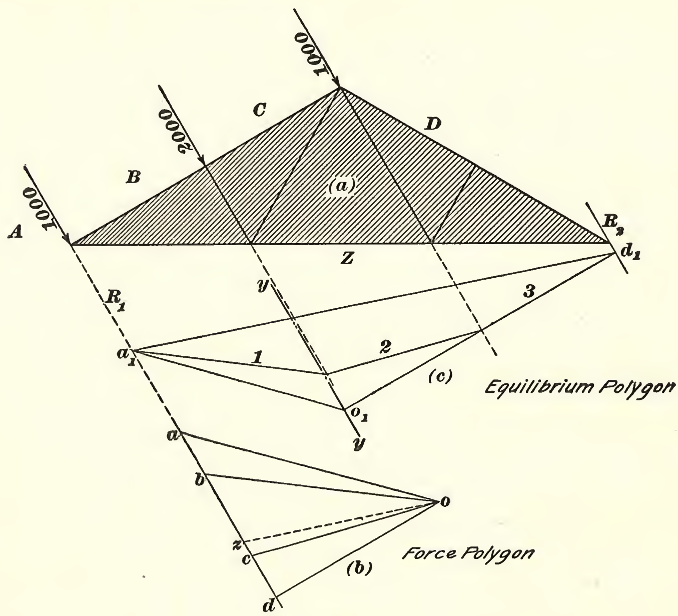


FIG. 33

35. In Fig. 33 (a) is shown the same truss but supporting a wind load only on its left-hand portion. These wind stresses act parallel with each other and normal to the slope of the roof, so that if the truss is fixed at the ends, the reactions R_1 and R_2 will coincide in direction with these forces.

To determine the amount of the wind reactions R_1 and R_2, construct the force polygon (b). Lay off the load line from a to d and represent to the proper scale the forces $a\,b$, $b\,c$, and $c\,d$, which correspond with the wind loads $A\,B$, $B\,C$, and

CD. Connect any point or pole o with the several divisions on the load line, thus obtaining the oblique lines $a\,o$, $b\,o$, $c\,o$, and $d\,o$. Commence the equilibrium polygon (c) at any point on the left-hand wind reaction, as a_1, and draw line *1* parallel with $b\,o$, line *2* parallel with $c\,o$, and line *3* parallel with $d\,o$, this last line intersecting the reaction R_2 at the point d_1; $a_1\,o_1$ is drawn parallel with $a\,o$ and line *3* is extended until it intersects $a_1\,o_1$ at the point o_1. By connecting the points d_1 and a_1 in the equilibrium polygon and drawing in the force polygon, from o, a line parallel with the line just described, the point z is located and the reaction R_2 is found by measuring $d\,z$, while the amount of R_1 is known by scaling $z\,a$.

It will be noticed in drawing this diagram that the lines *1*, *2*, and *3* were drawn between the lines of action of the forces *A B*, *B C*, and *CD*, and that the force *3* coincides with the reaction component $d_1\,o_1$. The reason that the last force *2* does not close on the line of action of the reaction R_2 is because there is no force on the right-hand portion of the truss. If there had been a force corresponding with *A B*, the reaction component $d_1\,o_1$ would have been distinct from the force *3* and the diagram to the right would have been similar to the portion at the left. On being measured, $d\,z$ and $z\,a$ are found to equal, respectively, 1,350 and 2,650 pounds.

The total reaction for both the wind and dead loads is, of course, the resultant of the two right-hand and the two left-hand reactions already found. These resultant reactions it will be noted are not parallel.

36. In Fig. 34 is shown the method just described applied to a roof truss supporting a vertical load. The truss is considered fixed at the ends, but the amount of the reactions R_1 and R_2 is not known, so that these will be determined by the graphical method. As in the previous cases, the force polygon is laid out as shown at (b). Commence this diagram by laying off the load line, which extends from a to h. In designating the several forces making up this

line, they were for convenience taken in sequence, that is, in the order in which they occur around the truss. Since the forces in the load line are coincident with regard to direction, their resultant will lie along the load line, and will extend from a to h. This line will represent, by scale, the

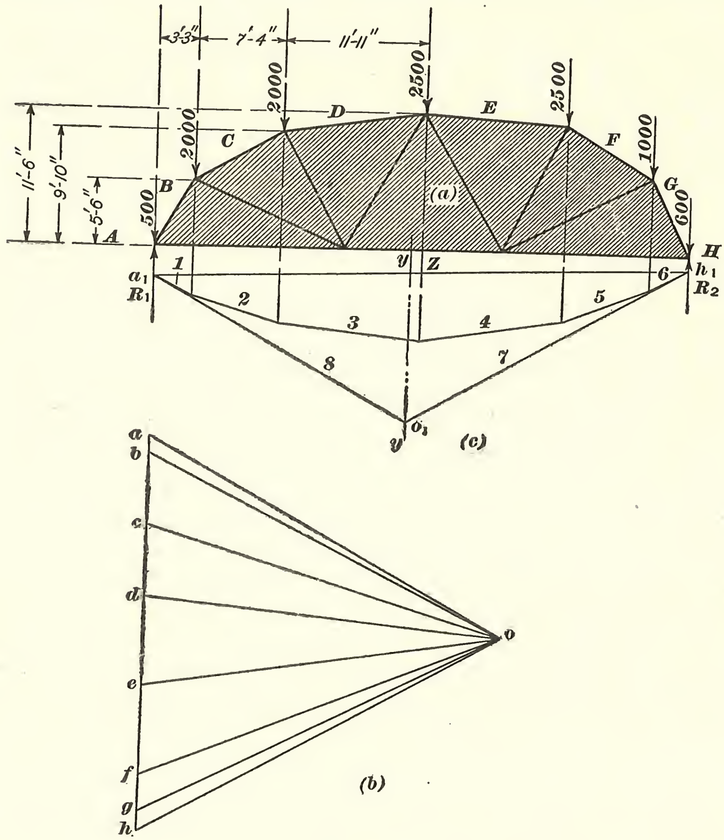


FIG. 34

sum of the reactions R_1 and R_2 and its direction coincides with the direction of the reactions. It is then in order to determine what percentage of the length of this line will represent R_1 and what percentage will represent R_2; to do this, the equilibrium polygon (c) must be drawn.

Before commencing this polygon, connect some pole or point o, in the force polygon, with each point on the load line, as shown by the oblique lines $a\,o$, $b\,o$, $c\,o$, $d\,o$, $e\,o$, etc.; also, extend the line of action of each of the forces acting on the truss, as shown by the dotted lines. Start the equilibrium polygon from any point a_1 on the line of action of R_1 by drawing line *1* parallel with $b\,o$ in the force polygon. This line should be drawn until it intersects the line of action of $B\,C$. The line *2* is drawn from this intersection in a direction parallel with $c\,o$ in the force polygon and of such a length that it intersects the line of action of the force $C\,D$. From this intersection, line *3* is drawn parallel with $d\,o$ in the force polygon until it intersects $D\,E$ extended. From this intersection, line *4* is drawn parallel with $e\,o$, and where this line intersects the force $E\,F$ extended, line *5* is drawn parallel with $f\,o$ intersecting $F\,G$, and from this point line *6* is drawn parallel with $g\,o$ intersecting the reaction R_2 at h_1. From this point, line *7* is drawn parallel with $h\,o$ in the force polygon and intersecting at o_1 line *8* drawn from a_1 parallel with $a\,o$ in the force polygon.

37. The reactions for the several frames described in connection with Figs. 32, 33, and 34 can be found from the equilibrium polygon. In each case a line $y\,y$ drawn through the point o_1 and parallel with the reactions will divide the cord connecting the points on the two reactions into two such parts as to inversely represent the amount of the reactions; for instance, in the equilibrium polygon in Fig. 32, the line $y\,y$ divides the line $a_1\,f_1$ into two such parts that if the entire line $a_1\,f_1$ is considered as representing the sum of the reactions R_1 and R_2, $y\,f_1$ will equal R_1 and $a_1\,y$ will equal R_2. Again, in the equilibrium polygon, Fig. 33, the line $y\,y$ drawn through o_1 parallel with the load line divides the line $a_1\,d_1$ into two such parts that the portion to the right equals the amount of the reaction R_1, while the portion to the left equals the amount of the reaction R_2, considering that the entire length of the line represents the sum of the reactions. Again, in Fig. 34, the line $y\,y$ coincides with

the direction of the reactions and divides $a_1 h_1$ into such proportion of parts that $y h_1 : a_1 h_1 = R_1 : R_1 + R_2$, and $a_1 y : a_1 h_1 = R_2 : R_1 + R_2$.

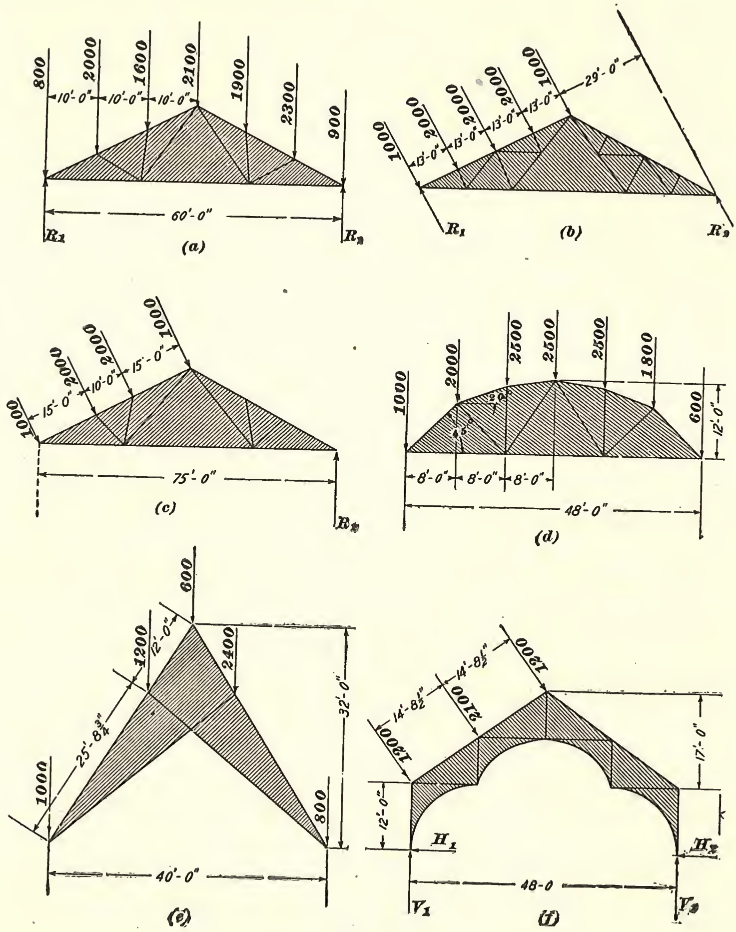


FIG. 35

EXAMPLES FOR PRACTICE

1. Determine, by calculation, the amount of the reactions R_1 and R_2 for the frame shown in Fig. 35 (*a*).

Ans. $\begin{cases} R_1 = 5{,}600 \\ R_2 = 6{,}000 \end{cases}$

2. Calculate the amount of the reactions R_1 and R_2 of the frame shown in Fig. 35 (*b*).

Ans. $\begin{cases} R_1 = 5{,}432 \\ R_2 = 2{,}568 \end{cases}$

3. Find the amount of R_2, by calculation, for the frame in Fig. 35 (*c*), and determine the amount and direction of R_1 by drawing the polygon of external forces. The right-hand end of the truss is on rollers, and the left-hand end is fixed.

Ans. $\begin{cases} R_1 = 4{,}566 \\ R_2 = 1{,}600 \end{cases}$

4. Determine, by the graphical method, the amount of the reactions for a frame loaded as shown in Fig. 35 (*d*).

Ans. $\begin{cases} R_1 = 6{,}717 \\ R_2 = 6{,}183 \end{cases}$

5. Employ the graphical method in determining the reactions for the scissor truss, shown in Fig. 35 (*e*), when loaded with vertical loads as designated.

Ans. $\begin{cases} R_1 = 2{,}900 \\ R_2 = 3{,}100 \end{cases}$

6. In Fig. 35 (*f*) is shown a hammer-beam roof truss. Find the vertical and horizontal components of the resultant reactions at both ends, graphically.

Ans. $\begin{cases} H_1 = 740 \\ H_2 = 1{,}600 \\ V_1 = 860 \\ V_2 = 1{,}875 \end{cases}$

DETERMINATION OF INTERNAL STRESSES

METHOD BY SOLUTION OF JOINTS

38. Maxwell's Method.—The method generally adopted for determining the internal stresses of a framed structure is known as **Maxwell's method,** named after the celebrated Scotch scientist, J. Clerk Maxwell. After the reactions have been found, either analytically or graphically, by the method given in the preceding pages, the force polygon for each joint in the frame is found. The assembled polygons of forces, which are laid out for each joint, form the stress diagram, which it is usual to commence at the left-hand end of the frame and work around consecutive

joints. It is not always possible, however, to follow this order, for often the joint in sequence contains more than two unknown stresses, in which case it becomes necessary to analyze some other joint in the frame around which there are only two unknown forces. In working around the frame, it is usual to begin at the left-hand end of the truss and read off and lay out the stresses around the joint in the same direction as the movement of the hands of a watch. It would be possible to work around each joint in the opposite direction, that is, in a direction opposite to the movement of the hands of a watch. But whatever may be the direction of the first joint analyzed it must be followed in analyzing the others.

In order that this important method, which will be used extensively in the application of graphical statics in analyzing the internal stresses of a framed structure may be thoroughly understood, the example illustrated in Fig. 36 will be assumed. In this figure is shown the frame and vertical-load stress diagrams for a quadrilateral truss. The first procedure in the analysis of any frame by the graphical method is to determine all the external forces. In this instance the loads are given, and since the truss is symmetrically loaded, the reactions are each equal to one-half of the loads on the truss, so that R_1 and R_2 are each equal in amount to 2,100 pounds, and are designated, respectively, as AZ and FZ, when the notation explained in Art. **29** is employed. The polygon of external forces, since all of these forces are coincident in direction, is contained within a straight line, which is represented in the stress diagram from a to f. This load line is determined by laying off the force ab equal in amount to AB in the frame diagram; likewise, bc equal to BC; and following around the truss in this manner until the point f has been located. When f has been determined, measure upwards on the load line an amount equal to R_2, or 2,100 pounds to scale, in this manner locating the point z, so that fz represents, by scale, the amount of the reaction R_2. The distance from z to a should then measure, by scale, an amount equal to R_1. The polygon of external forces will now

be a closed figure and will extend, beginning at the left-hand foot of the truss and taking the forces in consecutive order toward the right, from the point *a* in the stress diagram to *b*,

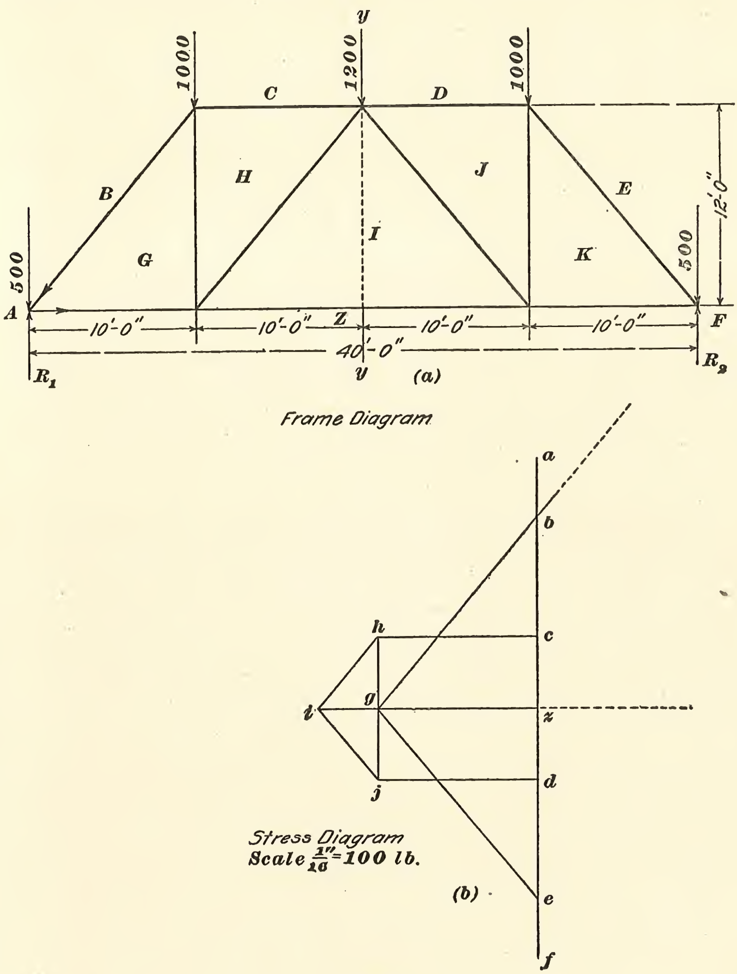


FIG. 36

from *b* to *c*, from *c* to *d*, from *d* to *e*, from *e* to *f*, and from *f* to *z* upwards, corresponding with the direction of R_2, and from *z* upwards to *a*, likewise corresponding in direction to R_1.

the last line thus closing the polygon. It is important to remember that before the stress diagram can be attempted, the polygon of external forces must be completed. If this polygon is incorrectly drawn or does not close, the diagram cannot be correctly laid out.

Having determined the polygon of external forces, the joint *A B G Z* in the frame diagram can be analyzed. At this joint in the frame there are only too unknown stresses, *B G* and *G Z*. The known stresses, *A B* and *Z A*, are already laid out in the stress diagram and are correspondingly lettered. To find the stresses in the members *B G* and *G Z*, draw an indefinite line from the point *b* in the stress diagram parallel with *B G* in the frame diagram, and from *z* draw a line parallel with *G Z*. The point of intersection of these two lines will be the point *g* and the polygon for the system of forces meeting at the extreme left-hand joint of the truss will read, in the stress diagram, from *a* to *b*, from *b* to *g*, from *g* to *z*, and from *z* back to *a*, the point of commencement.

39. Determination of the Kind of Stress.—In drawing the stress diagram the question naturally arises as to the direction in which these lines should be drawn, since a line might be drawn from *b* upwards, as shown by the dotted line, being still parallel with *B G* in the frame diagram; and in like manner the line *z g* might be drawn in a direction from *z* toward the right instead of toward the left. It is quite evident, however, that if the lines were drawn as suggested and shown dotted, they would not intersect and the polygon for the system of forces around the left-hand joint of the frame would not close. It is clear, therefore, that the lines that represent the stresses in the members *B G* and *G Z* can be drawn only in one direction and the direction in which they are read determines the kind of stress that exists in the member. For instance, in drawing the polygon for the stresses around the joint *A B G Z*, the force *a b* in the stress diagram is downwards. The stress from *b* to *g* is likewise downwards and may be designated by an arrow on *B G* in the frame diagram; *g z* is to the right,

likewise shown by the arrow on this member in the frame diagram, the final line of the polygon being upwards from z to a and thus representing the amount of the reaction R_1.

As the stresses are read, their direction should be marked with arrows on the frame diagram; this shows the direction of the stress and designates whether it is compression or tension. Forces acting away from a joint are always tensile stresses; those acting toward a joint are always compressive stresses.

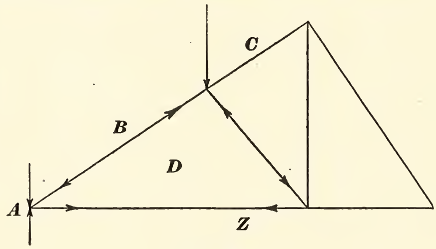


Fig. 37

If there is tension at one end of a member, it is evident there must be an equal amount of tension at the other end; if there is compression at one end of a member, there is an equal amount at the other.

An easy way to remember whether the arrows designate compression or tension by their direction, as shown on the members in a frame diagram, may be seen by referring to Figs. 37 and 38. The member DZ, Fig. 37, is a tension member; the arrows point away from the joints and toward each other, and resemble the form of an elastic material, stretched, as shown at A, Fig. 38; while in the compression member BD, the arrows act toward the joints, or away from each other, and resemble the form that a plastic material assumes on being compressed, as shown at B, Fig. 38.

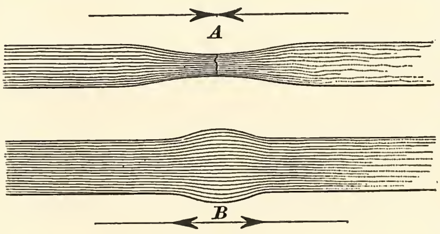


Fig. 38

40. Proceeding with the solution of the frame diagram shown in Fig. 36, the joint $BCHG$ may next be analyzed. Around this joint the only unknown forces are the stresses in the members CH and HG. To find the amount of these stresses, draw a line in the stress diagram from the point c

parallel with CH, and from g draw a line upwards parallel with GH; the point of intersection will be h. The polygon of forces about the joint under consideration will then extend in the stress diagram from b to c, from c to h, from h to g, from g back to the starting point b. The joint $CDJIH$ cannot as yet be analyzed, for there are three unknown forces represented by the members DJ, JI, and IH, but the joint marked $GHIZ$ can be analyzed, for the only unknown stresses exist in the members HI and IZ. To analyze this joint, draw from the point h, already determined, an indefinite line parallel with HI in the frame diagram, and from z a line parallel with IZ, intersecting this first line at i; thus, the polygon of forces around the joint $GHIZ$ is complete and extends in the stress diagram from g to h, from h to i, from i to z, and from z back to g. In analyzing this last joint, the stress in the member hi was determined, so that there now exist only two unknown forces at the joint $CDJIH$. By analyzing this joint, the polygon of external forces will be found to extend in the stress diagram from c to d, from d to j, from j to i, from i to h, and from h back to the point c, and it will be found that the stress diagram will begin to repeat on the lower side of the line zi. If the frame diagram is further analyzed, it will be found that the stress diagram is balanced on the line zi and is symmetrical about this axis, showing that the truss is symmetrically loaded. Since this is the case, only one-half of the stress diagram need be drawn, for the stresses in the members on each side of the center line of the frame diagram marked yy are the same, owing to the fact that the truss is symmetrically loaded.

The stresses in the several members may now be determined by measuring their respective lines in the stress diagram, and a tabulation of these stresses will give a convenient table for designing the several members to sustain tensile and compressive stresses.

GRAPHICAL STATICS BY THE METHOD OF SECTIONS

41. Another method of drawing the stress diagrams for any framed structure is by the **method of sections,** and while it differs somewhat from the method just explained, the results are the same. In general, this method consists of passing a plane through any section of the frame in such a manner that it will intersect only two members in which the forces are unknown, these two unknown stresses being obtained by completing the polygon of forces necessary to produce equilibrium in that portion of the frame lying to one side or the other of the plane. Since the method is more readily explained by an application of its principles to an example, the frame diagram of a quadrangular truss, shown in Fig. 39, will be analyzed. It is necessary to lay off the load line in the same manner as in the analysis of the joints of the frame by considering the forces about the joints. The load line for the frame diagram, Fig. 39, will then be a vertical line extending from a to b, b to c, etc., from f back to z, giving the reaction R_2, and from z to the point a, the length of the latter line being equal to the reaction R_1. Commencing at the left-hand joint of the frame diagram, pass a plane through the frame, cutting the several members along the line $a\,b$. The portion of the frame to the left of this plane is held in equilibrium by the reaction R_1, the force AB, and the stresses in the members BG and GZ, the only two unknown forces about the joint consisting of the latter. These four forces form a system that acts on the portion of the truss to the left of the line $a\,b$, as is shown in (a). The direction of the unknown forces being known, they may be drawn in the stress diagram as shown, and their point of intersection will be g. Another plane is now passed through the frame, cutting the members along the line $c\,d$ and the left-hand portion of the truss is held in equilibrium by the reaction R_1, the loads AB and BC, and the stresses CH, HG, GZ. The portion of the frame held in equilibrium by these forces to the left of the line $c\,d$ is shown in (b). The only two unknown forces acting on the left-hand portion

of the frame are CH and HG, for all the other forces have been shown in the stress diagram. To find these forces, then, and close the polygon for this portion of the frame, all

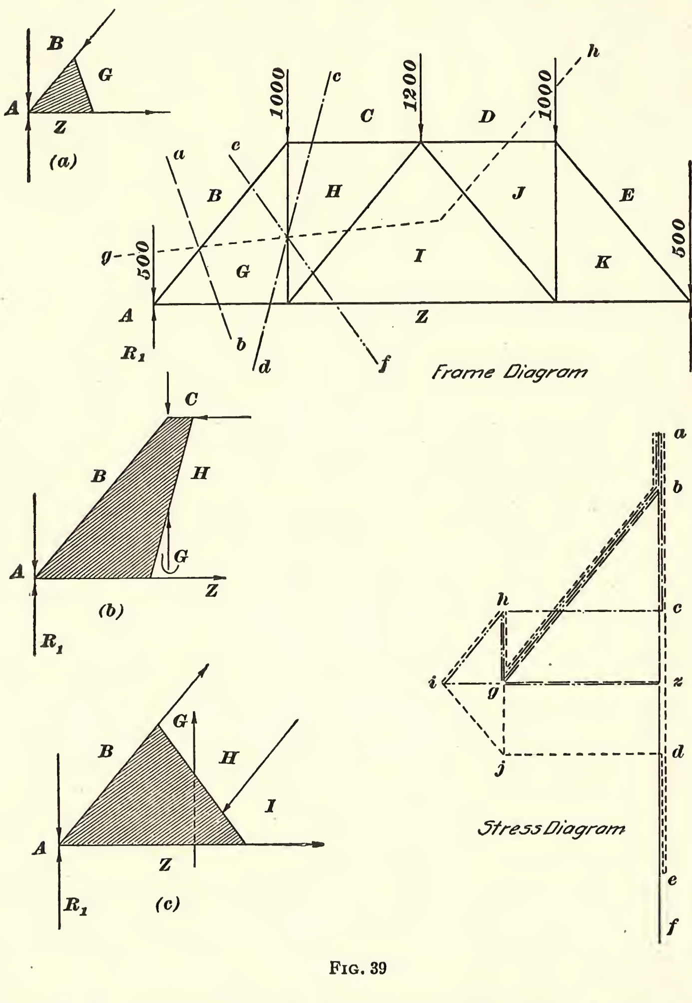


FIG. 39

that it is necessary to do is to draw the lines ch and hg in the stress diagram, thus locating the point h. Again, assume that a plane is passed through the frame along the line ef;

the left-hand portion of the frame will be held in equilibrium by the forces shown in (*c*), which are the reaction R_1, the load AB, and the stresses in the members BG, GH, HI, and IZ. The two unknown stresses are HI and IZ, and their amounts are determined when the lines hi and iz have been drawn in the stress diagram, and the polygon of forces for the left-hand portion of the truss has been completed. A separate line has been used in the stress diagram to represent the polygon of forces in each case, in order that the solution of each section may be clear.

This system can be continued throughout the truss, and it will be found on comparison of the frame diagram with the stress diagram, that any plane, such as gh, will cut the truss in such a manner that each portion will be in equilibrium; and that when the stress diagram has been entirely completed the polygon will close for either portion of the frame diagram cut by the plane. This system of graphical statics is preferred by many, but the other is generally practiced.

GRAPHICAL ANALYSIS OF STRESSES

(PART 1)

EXAMINATION QUESTIONS

(1) (*a*) What is an external force? (*b*) What is an internal force? (*c*) Explain what is understood by equilibrium.

(2) (*a*) What is complete equilibrium? (*b*) What is understood by the resultant of any system of forces?

(3) Describe the fundamental principles of the relations between force and motion.

(4) Determine the amount of the resultant of the two forces in Fig. I by describing the parallelogram of forces.

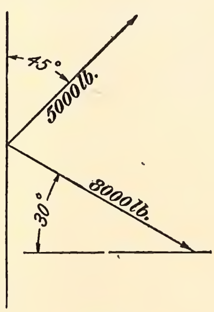


FIG. I

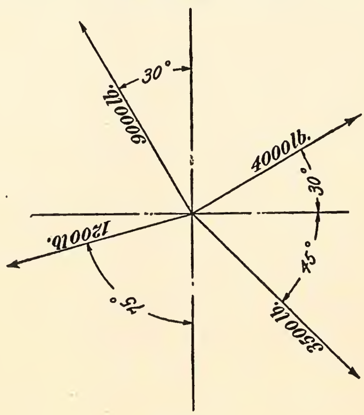


FIG. II

(5) Determine the resultant of the several concurrent forces shown in Fig. II.

(6) Determine the horizontal and vertical components of the oblique force shown in Fig. III.

(7) The weight of a coping stone resting on the inclined top of a wall is 1,000 pounds, the angle of inclination to the horizontal being 30°. Provided that the stone is set in cement mortar and no friction exists between the coping and wall, what force does the stone exert perpendicular to the inclined top of the wall and what force does it exert parallel with the incline tending to slide it downwards?

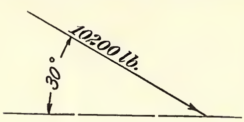


FIG. III

(8) State, by a rule, the conditions that must exist for any number of concurrent forces acting at the joint of a framed structure to be in equilibrium. Also, express the rule by an equation.

(9) A girder 33 feet long is loaded with three concentrated loads located at distances from the left-hand end equal to 10, 15, and 21 feet, respectively. The first concentrated load is 5,000 pounds, the second 8,000 pounds, and the third 12,000 pounds. Determine by the force and equilibrium polygons the reactions R_1 and R_2 at the left- and right-hand ends, respectively.

Ans. $\begin{cases} R_1 = 12{,}150 \\ R_2 = 12{,}850 \end{cases}$

(10) A girder supported at both ends, as designated in Fig. IV, is required to sustain the thrust of two oblique shores W_1 and W_2. Determine the amount and direction of the reactions R_1 and R_2.

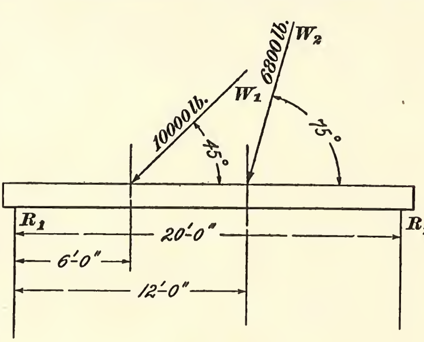


FIG. IV

Ans. $\begin{cases} R_1 = 9{,}000 \\ R_2 = 7{,}300 \\ \text{Direction: } 33° \text{ with the vertical} \end{cases}$

(11) A shed roof is supported on two light brick walls as shown in Fig. V. Besides supporting the vertical loads on the rafter members, as designated, the truss is required to

support an oblique load of 1,000 pounds from belting and shafting attached at the point *c*, the general direction of the pull being 60° with the horizontal, as shown in the figure. Determine, by the graphical method, the direction and amounts of the reactions R_1 and R_2 on the left- and right-hand walls.

Ans. $\begin{cases} R_1 = 2{,}100 \\ R_2 = 1{,}800 \\ \text{Direction: } 7^\circ\ 15' \text{ with the vertical} \end{cases}$

(12) Explain, with figures: (*a*) If the amount of three forces is known, how can their directions be determined? (*b*) How can the amounts of two unknown forces be determined when their direction and the amount and direction of the third force are known? (*c*) When the directions and amounts of two forces are known, how may the direction and amount of the third force be determined?

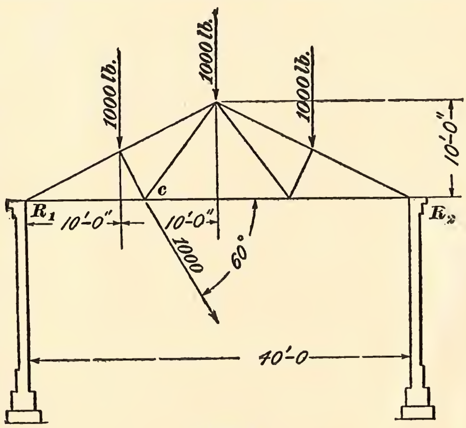


Fig. V

(13) (*a*) Explain, with sketch if necessary, what is understood by a frame diagram. (*b*) Explain, with sketch, what is meant by a stress diagram.

(14) Explain the usual notation in lettering a stress diagram.

(15) What is essentially the first step in drawing a stress diagram?

(16) Considering that the direction of R_2 is horizontal, determine, by calculation, the amounts of the reactions at R_1 and R_2, also the direction of R_1, for the overhanging roof shown in Fig. VI.

Ans. $\begin{cases} R_1 = 4{,}750 \\ \text{Direction of } R_1 : 42^\circ\ 26' \\ R_2 = 3{,}200 \end{cases}$

(17) Check the results of question 16 by the graphical method.

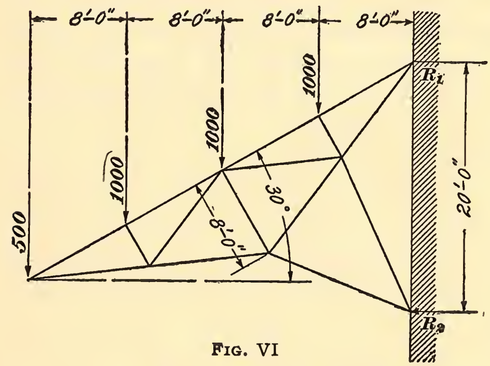


Fig. VI

(18) Employ the graphical method for determining the amounts of reactions R_1, R_2 and the direction of R_1 for the truss shown in Fig. VII. The right-hand end of the truss is supported on rollers.

Ans. $\begin{cases} R_1 = 18{,}000 \\ \text{Direction of } R_1 : 7^\circ\ 45' \text{ with vertical} \\ R_2 = 18{,}375 \end{cases}$

(19) Describe, in detail, by means of a simple example, the method of drawing the stress diagram for a framed structure by Maxwell's method.

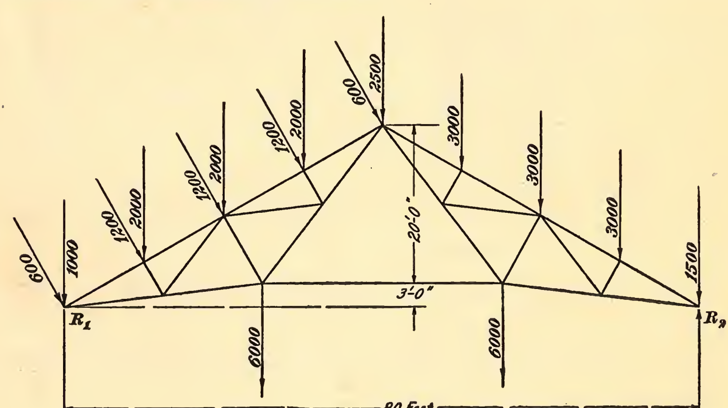


Fig. VII

(20) Describe, in detail, how the stresses in the members of a frame are determined by the method of sections.

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